Section-I: General Ability

1. Hema's age is 5 years more than twice Hari's age. Suresh's age is 13 years less than 10 times Hari's age. If Suresh is 3 times as old as Hema, how old is Hema?

(A) 14 (B) 17 (C) 18 (D) 19

Key: (D)

Exp: Let Hari's age be 'x' years.

$$\left.\begin{array}{l}\text{Then Hema' age} = 2x+5 \\ \text{Suresh's age} = 10x-3\end{array}\right] \rightarrow (\text{Given})$$

Also given, Suresh is 3 times as old as Hema.

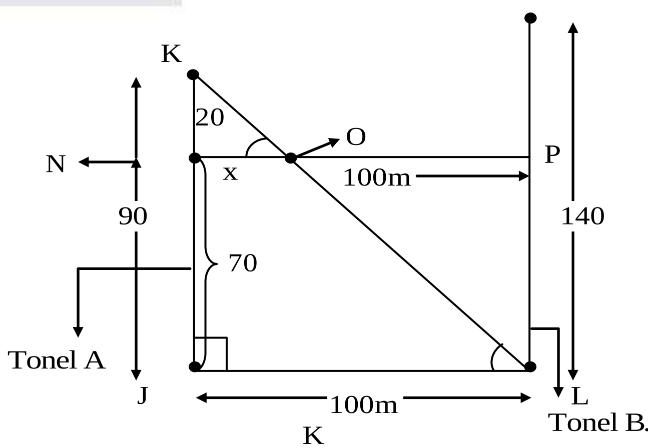
$$\text{i.e.}, 10x-13 = 3(2x+5)$$
$$\Rightarrow 10x-6x = 15+13$$
$$\Rightarrow 4x = 28$$
$$\Rightarrow x = 7$$
$$\therefore \text{Hema's age} = 2x+5 = 2(7)+5 = 19 \text{ years}$$

2. Tower A is 90m tall and tower B is 140 m tall. They are 100m apart. A horizontal skywalk connects the floors at 70m in both the towers. If a taut rope connects the top of tower A to the bottom of tower B, at what distance (in meters) from tower A will the rope intersect the skywalk?

Key: 22.22

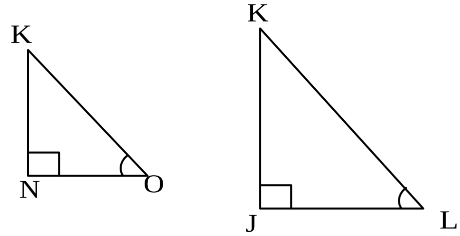
Exp: From the given data;

Clearly;



Δ^{le} KNO & Δ^{le}KJL are similar traingles; since

$$\left\{\begin{array}{l}\angle KNO = \angle KJO = 90^\circ \\ \angle NOK = \angle JLK. \quad [\because NP // JL] \\ \text{By AA similarity criterian;} \\ \Delta^{le}\text{ KNO \& }\Delta^{le}\text{KJL are similar}\end{array}\right\}$$



$[\angle N = \angle J = 90, \angle O = \angle L]$

$$\therefore \frac{NK}{KJ} = \frac{NO}{JL}$$
$$\Rightarrow \frac{20}{90} = \frac{x}{100} \; [\text{where } x = NO]$$
$$\Rightarrow x = \frac{100\times 20}{90} = 22.22 \text{ meters.}$$

3. The temperature T in a room varies as a function of the outside temperature T_0 and the number of persons in the room p, according to the relation $T=K(\Theta p+T_0)$, Where Θ and K are constants. What would be the value of Θ given the following data?

T_0	P	T
25	2	32.4
30	5	42.0

(A) 0.8 (B) 1.0 (C) 2.0 (D) 10.0

Key: (B)

Exp: Given,

$T=K(\theta P+T_o)$; where θ & K are constants $\rightarrow (1)$

From the given table; $T_o=25; P=2; T=32.4$

&

$T_o=30; P=5; T=42.0$

From $(1) \Rightarrow$ $32.4=K(\theta 2+25)\rightarrow(2)$

$42.0=K(\theta 5+30)\rightarrow(3)$

$$(2)\times 5-(3)\times 2 \Rightarrow 162=10K\theta+125K$$

$$84=10K\theta+60K$$

$$\underline{(-)\quad(-)\qquad(-)}$$

$$\Rightarrow 78=65K$$

$$\Rightarrow K=78/65=1.2$$

$$\therefore \text{From}(2); 32.4=K(\theta 2+25)$$

$$\Rightarrow 32.4=1.2(\theta 2+25)$$

$$\Rightarrow 32.4=2.4\theta+30$$

$$\Rightarrow 2.4\theta=2.4$$

$$\Rightarrow \theta=\frac{2.4}{2.4}=1.$$

4. "The driver applied the ___ as soon as she approached the hotel where she wanted to take a ___."

The words that best fill the blanks in the above sentence are

(A) brake, break (B) break, break (C) brake, brake (D) break, brake

Key: (A)

5. "It is no surprise that every society has had codes of behaviour; however, the nature of these codes is often __________."

The word that best fills the blank in the above sentence is

(A) unpredictable (B) simple (C) expected (D) strict

Key: (A)

6. Each of the letters arranged as below represents a unique integer from 1 to 9. The letters are positioned in the figure such that (A×B×C), (B×G×E) and (D×E×F) are equal. Which integer among the following choices cannot be represented by the letters A, B, C, D, E, F or G?

A		D
B	G	E
C		F

(A) 4 (B) 5 (C) 6 (D) 9

Key: (B)

Exp: Consider

$A=1; B=9; C=8$

$G=2$

$D=6; E=4; F=3.$

Then $A\times B\times C=1\times 9\times 8=72$

$B\times G\times E=9\times 2\times 4=72$

$D\times E\times F=6\times 4\times 3=72$

$\therefore E=4; D=6; B=9$

The integer '5' cannot be represented by the letters A, B, C,D, E,F,G.

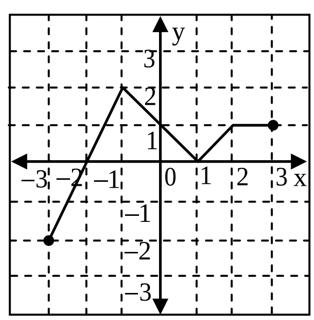
7. Which of the following function (s) is an accurate description of the graph for the range (s) indicated?

(i) $y=2x+4$ for $-3\le x\le -1$

(ii) $y=|x-1|$ for $-1\le x\le 2$

(iii) $y=\left||\,x\,|\,x-1\right|$ for $-1\le x\le 2$

(iv) $y=1$ for $2\le x\le 3$



(A) (i), (ii) and (iii) only (B) (i), (ii) and (iv) only

(C) (i) and (iv) only (D) (ii) and (iv) only

Key: (B)

Exp: From the graph; y =1 for $2 \le x \le 3$.

$\Rightarrow (iv)$ is correct.

From the graph we can see that

$y = 2x + 4$ for $-3 \le x \le -1$

$[\text{If } x = -3; y = -2 \ \& \ \text{If } x = -1; y = 2]$

$\therefore (i)$ is correct

From the graph;

if $x = 1, y = 0; \quad x = -1, y = 2; \quad x = 2, y = 1$

i.e, $y = |x - 1|$ for $-1 \le x \le 2$

$\therefore (ii)$ is correct.

But (iii) is not correct; since at $x = -1; y = 0$.

But from the graph at $x = -1; y = 2$.

8. Consider a sequence of numbers $a_1, a_2, a_3, \ldots, a_n$ where $a_n = \frac{1}{n} - \frac{1}{n+2}$, for each integer n>0. What is the sum of the first 50 terms?

(A) $\left(1+\frac{1}{2}\right)-\frac{1}{50}$

(B) $\left(1+\frac{1}{2}\right)+\frac{1}{50}$

(C) $\left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$

(D) $1-\left(\frac{1}{51}+\frac{1}{52}\right)$

Key: (C)

Exp: Given $a_n = \frac{1}{n} - \frac{1}{n+2}; n > 0;$ where 'n' is integer.

$$\therefore a_1 = \underbrace{\frac{1}{1} - \frac{1}{3}}_{+2}; a_2 = \underbrace{\frac{1}{2} - \frac{1}{4}}_{+2}; a_3 = \underbrace{\frac{1}{3} - \frac{1}{5}}_{+2}; \ldots; a_{50} = \underbrace{\frac{1}{50} - \frac{1}{52}}_{+2}$$

$\therefore$ sum of the first 50 terms $= a_1 + a_2 + a_3 + \ldots + a_{50}$

$$= \left(\frac{1}{1}-\frac{1}{3}\right)\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)\ldots\ldots+\left(\frac{1}{48}-\frac{1}{50}\right)$$

$$+\left(\frac{1}{49}-\frac{1}{51}\right)\ldots\ldots+\left(\frac{1}{50}-\frac{1}{52}\right)$$

$$= \left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)\left[\because \frac{1}{49} \text{ also get cancell with } +\left(\frac{1}{47}-\frac{1}{49}\right)\right]$$

$$\therefore \text{sum of the first 50 terms} = \left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$$

9. The price of a wire made of superalloy material is proportional to the square of its length. The price of 10m length of the wire is Rs. 1600. What would be the total price (in RS.) of two wires of lengths 4m and 6m?

(A) 768 (B) 832 (C) 1440 (D) 1600

Key: (B)

Exp: Let us assume that,

Length of wire = x m & price of wire Rs p/meter

Given $P \alpha x^2$.

$\Rightarrow P = kx^2 \rightarrow (1)$

Given; If $x = 10m$; then $P = 1600$

From $(1); 1600 = k(10)^2$

$\Rightarrow K = 16$

$\therefore P = 16x^2 \left[\because \text{form } (1)\right]$

If $x = 4$; then $p = 16 \times 4^2 = 16 \times 16 = 256 \rightarrow (1)$

If $x = 6$; then $p = 16 \times 6^2 = 16 \times 36 = 576 \rightarrow (2)$

$(1) + (2) \Rightarrow$ Total price $= 256 + 576 = 832.$

10 A fruit seller sold a basket of fruits at 12.5% loss. Had he sold it for Rs. 108 more, he would have made a 10% gain. What is the loss in Rupees incurred by the fruit seller?

(A) 48 (B) 52 (C) 60 (D) 108

Key: (C)

Exp: $CP \times 12.5\% = Loss \Rightarrow CP \times 12.5\% = CP - SP \rightarrow (1)$

$CP \times 10\% = Gain \Rightarrow CP \times 10\% = (SP + 108) - CP \rightarrow (2)$

$(1) + (2) \Rightarrow CP[22.5\%] = 108$

$$CP = \frac{108}{22.5\%}$$

$$\therefore Loss = \frac{108}{22.5\%} \times 12.5\% = 60. (\because CP \times 12.5\% = Loss)$$

Section–II: Civil Engineering

1. The percent reduction in the bearing capacity of a strip footing resting on sand under flooding condition (water level at the base of the footing) when compared to the situation where the water level is at a depth much greater than the width of footing, is approximately

(A) 0 (B) 25 (C) 50 (D) 100

Key: (C)

Exp: $q_u = C.N_C + (\gamma.D_f).N_q + \frac{1}{2}B\gamma N_\gamma$ (Strip footing) for footing on sandy ground,

$C = 0 ; D_f = 0$

$\Rightarrow q_u = \frac{1}{2}B\gamma N_\gamma$ (when water table is at a depth greater than width (B) of footing)

When water table comes at ground level than $\gamma = \gamma'$

$q_u = \frac{1}{2}B\gamma' N_\gamma$

$\therefore \gamma' \simeq \frac{\gamma}{2} \Rightarrow q_u = \frac{1}{4}B\gamma N_\gamma$

$$\Rightarrow \text{\% reduction in } q_u = \frac{\frac{1}{2}B\gamma N_\gamma - \frac{1}{4}B\gamma N_\gamma}{\frac{1}{2}B\gamma N_\gamma} \times 100\,\% = 50\%$$

2. A column of height h with a rectangular cross section of size a ×2a has a buckling load of P. If the cross section is changed to 0.5a×3a and its height changed to 1.5h, the buckling load of the redesigned column will be

(A) P/12 (B) P/4 (C) P/2 (D) 3P/4

Key: (A)

Exp: Buckling load , $P_u = \frac{\pi^2 EI_{min}}{L_{eff}^2}$

Given; $P = \frac{\pi^2 E\left(a^3 \times 2a/12\right)}{h^2} = \frac{\pi^2 Ea^4}{6h^2}$

Buckling load of redesigned column $= \dfrac{\pi^2 E \dfrac{\left((0.5a)^3 \times 3a\right)}{12}}{(1.5h)^2} = \frac{\pi^2 Ea^4}{72h^2} = \frac{P}{12}$

3. A steel column o ISHB 350 @ 72.4 kg/m is subjected to a factored axial compressive load of 2000kN. The load is transferred to a concrete pedestal of grade M20 through a square base plate. Consider bearing strength of concrete as $0.45f_{ck}$, where f_{ck} is the characteristic strength of concrete.

Using limit state method and neglecting the self weight of base plate and steel column, the length of a side of the base plate to be provided is

(A) 39cm (B) 42cm (C) 45cm (D) 48cm

Key: (D)

Exp: Bearing stress on concrete $= \dfrac{\text{Factored load}}{\text{Area of plate}}$

$$\Rightarrow 0.45 f_{ck} = \frac{2000 \times 1000\,\text{N}}{x^2} \left[x = \text{size of squre plate}\right]$$

$$\Rightarrow x = \sqrt{\frac{2\times 10^6}{0.45 \times 20}}$$

$$\Rightarrow x = 471.4\text{mm} \Rightarrow x = 47.14 \simeq 48\text{cm}$$

4. A 1:50 model of a spillway is to be tested in the laboratory. The discharge in the prototype spillway is 1000m^3/s. The corresponding discharge (in m^3/s, up to two decimal places) to be maintained in the model, neglecting variation in acceleration due to gravity, is ________.

Key: 0.0565

Exp: For flow over spillway, froude's model law is and :

$$\Rightarrow F_m = F_p$$

$$\Rightarrow \frac{V_m}{\sqrt{gD_m}} = \frac{V_p}{\sqrt{gD_p}} \Rightarrow \frac{V_m^2}{D_m} = \frac{V_p^2}{D_p}$$

$$\Rightarrow \frac{D_m^4 V_m^2}{D_m^5} = \frac{V_p^2 D_p^4}{D_p^5}$$

$$\Rightarrow \frac{Q_p}{Q_m} = \left(\frac{D_p}{D_m}\right)^{5/2} \Rightarrow Q_m = \left(\frac{1}{50}\right)^{5/2} \times 1000$$

$$Q_m = 0.0565\, m^3/s$$

5. A bitumen sample has been graded as VG30 as per IS : 73-2013. The '30' in the grade means that

(A) penetration of bitumen at 25°C is between 20 and 40

(B) viscosity of bitumen at 60°C is between 2400 and 3600 poise

(C) ductility of bitumen at 27°C is more than 30 cm

(D) elastic recovery of bitumen at 15°C is more than 30%

Key: (B)

6. For the given orthogonal matrix Q

$$Q=\begin{bmatrix}3/7 & 2/7 & 6/7\\ -6/7 & 3/7 & 2/7\\ 2/7 & 6/7 & -3/7\end{bmatrix}$$

The inverse is

(A) $\begin{bmatrix}3/7 & 2/7 & 6/7\\ -6/7 & 3/7 & 2/7\\ 2/7 & 6/7 & -3/7\end{bmatrix}$

(B) $\begin{bmatrix}-3/7 & -2/7 & -6/7\\ 6/7 & -3/7 & -2/7\\ -2/7 & -6/7 & 3/7\end{bmatrix}$

(C) $\begin{bmatrix}3/7 & -6/7 & 2/7\\ 2/7 & 3/7 & 6/7\\ 6/7 & 2/7 & -3/7\end{bmatrix}$

(D) $\begin{bmatrix}-3/7 & 6/7 & -2/7\\ -2/7 & -3/7 & -6/7\\ -6/7 & -2/7 & 3/7\end{bmatrix}$

Key: (C)

Exp: Given, G is orthogonal matrix.

For orthogonal matrix; $Q^{-1}=Q^{T}$

$$\Rightarrow Q^{-1}=\begin{bmatrix}3/7 & 2/7 & 6/7\\ -6/7 & 3/7 & 2/7\\ 2/7 & 6/7 & -3/7\end{bmatrix}^{T};\ \text{where } Q=\begin{bmatrix}3/7 & 2/7 & 6/7\\ -6/7 & 3/7 & 2/7\\ 2/7 & 6/7 & -3/7\end{bmatrix}$$

$$\Rightarrow Q^{-1}=\begin{bmatrix}3/7 & -6/7 & 2/7\\ 2/7 & 3/7 & 6/7\\ 6/7 & 2/7 & -3/7\end{bmatrix}.$$

7. The frequency distribution of the compressive strength of 20 concrete cube specimens is given in the table.

$f\,(\text{MPa})$	Number of specimens with compressive strength equal to f
23	4
28	2
22.5	5
31	5
29	4

If μ is the mean strength of the specimens and σ is the standard deviation, the number of specimens (out of 20) with compressive strength less than $\mu-3\sigma$ is _______

Key: (0)

Exp: mean strength $(\mu)=\dfrac{4\times23+2\times28+5\times22.5+5\times31+4\times29}{4+2+5+5+4}=\dfrac{531.5}{20}=26.575\,\text{MPa}$

$$\text{sandard deviation } \sigma = \sqrt{\frac{(23-26.575)^2\times4+(28-26.575)^2\times2+(22.5-26.575)^2\times5+(31-26.575)^2\times5+(29-26.575)^2\times4}{20-1}}$$

$$=3.697$$

$$\mu-3\sigma=26.575-3\times3697=15.487$$

No specimen is having compressive strength of 15.487 MPa (or nearer)

8. At the point x=0, the function f(x) $=x^3$ has
(A) local maximum
(B) local minimum
(C) both local maximum and minimum
(D) neither local maximum nor local minimum

Key: (D)

Exp: Given $f(x)=x^3$

We will find stationary points of f(x) by solving $f'(x)=0$

$\therefore f'(x)=0 \Rightarrow 3x^2=0 \Rightarrow x=0.$

$f''(x)=6x$

$\Rightarrow f''(0)=0$

$f'''(x)=6\neq0 \text{ at } x=0$

$\therefore x=0$ is a saddle point.

$\Rightarrow$ There is neither maximum nor minimum exists at x=0.

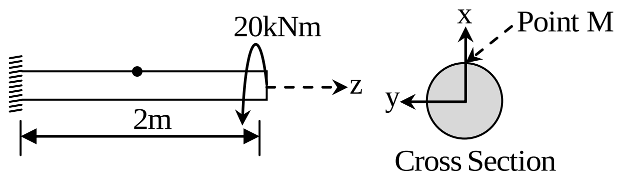
9. There are 20,000 vehicles operating in a city with an average annual travel of 12,000 km per vehicle. The NO_x emission rate is 2.0 g/km per vehicle. The total annual release of NO_x will be
(A) 4,80,000kg (B) 4,800kg (C) 480kg (D) 48kg

Key: (A)

Exp: Annual release of No_x by 1 vehicle $=\dfrac{12,000\times2}{1000}=24\text{kg}$

$\Rightarrow$ Annual release of all 20,000 vehicles $=24\times20,000=4,80,000$ kg

10. A solid circular beam with radius of 0.25 m and length of 2m is subjected to a twisting moment of 20kNm about the z-axis at the free end, which is the only load acting as shown in the figure. The shear stress component τ_{xy} at Point 'M' in the cross section of the beam at a distance of 1m from the fixed end is



(A) 0.0MPa (B) 0.51MPa (C) 0.815MPa (D) 2.0MPa

Key: (A)

Exp: Twisting moment = 20 kNm

By equation

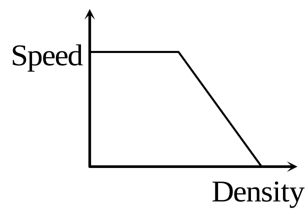
$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{\gamma}$$

$$\frac{\tau}{\gamma} = \frac{T}{J}$$

$$\tau_{max} = \frac{Tr}{J}$$

As the solid circular beam subjected to twisting moment about the Z-axis, then the shaft is subjected to shearing stresses in planes zy and xz only. The shear stress in xy plane is zero

11. The speed density relationship for a road section is shown in the figure.



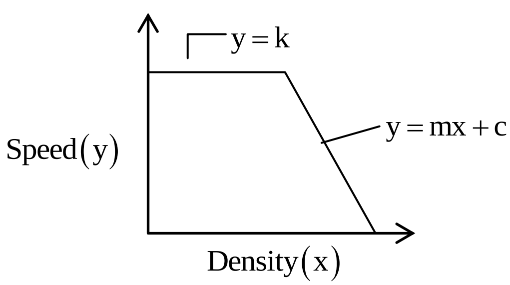
The shape of the flow-density relationship is

(A) piecewise linear (B) parabolic

(C) initially linear than parabolic (D) initially parabolic then linear

Key: (C)

Exp: Given



$\text{Flow} = \text{speed} \times \text{density}$

$= y \times x$

for $y = k$; $\text{Flow} = kx \Rightarrow \text{linear}$

for $y = mx + c$; $\text{Flow} = (mx + c)x$

$= mx^2 + c_x \Rightarrow \text{Parabalic}$

12. The width of a square footing and the diameter of circular footing are equal. If both the footings are placed on the surface of sandy soil, the ratio of the ultimate bearing capacity of circular footing to that of square footing will be

(A) 4/3 (B) 1 (C) 3/4 (D) 2/3

Key: (C)

Exp: $\left.q_u\right)$ circular $= 1.3CN_C + (\gamma D_f)N_q + 0.3\gamma DN\gamma$

For sandy soil & footing placed on the surface of soil,

$C = 0 \,\&\, D_f = 0$

$\Rightarrow \left.q_u\right)\text{circuler} = 0.3\gamma DN\gamma$

$\left.q_u\right)\text{square} = 1.3_C N_C + (\gamma D_f)N_q + 0.4\gamma BN_\gamma$

$= 0.4\gamma BN_\gamma$

$\because D = B \Rightarrow \dfrac{\left.q_u\right)\text{cicular}}{\left.q_u\right)\text{square}} = \dfrac{0.3\gamma BN_\gamma}{0.4\gamma BN_\gamma} = \dfrac{3}{4}$

13. A well designed signalized intersection is one in which the

(A) crossing conflicts are increased

(B) total delay is minimized

(C) cycle time is equal to the sum of red and green times in all phases

(D) cycle time is equal to the sum of red and yellow times in all phase

Key: (B)

Exp: For good signalised intersection, crossing conflicts should be lesser

$\rightarrow$ Vehicle delay is the most important parameter used by transportation professionals in evaluating the performance of a signalized intersection

$\rightarrow$ Minimum delay ensures lesser fuel loss, lesser congestion and lesser time loss of public ,

Hence, a well designed signalized intersection is one in which total delay in minimized

14. A flow field is given by $u=y^2$, $v=-xy$, $w=0$. Value of the z – component of the angular velocity (in radians per unit time, up to two decimal places) at the point (0,-1, 1) is _____

Key: (1.5)

Exp: $w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$$= \frac{1}{2}\left(\frac{\partial(-xy)}{\partial x} - \frac{\partial(y^2)}{\partial y}\right)$$

$$= \frac{1}{2}(-y-2y) = \frac{-3y}{2}$$

$$\text{At}(0,-1,1);\ w_z = \frac{-3\times(-1)}{2} = 1.5$$

15. In a shrinkage limit test, the volume and mass of a dry soil pat are found to be 50cm^3 and 88g, respectively. The specific gravity of the soil solids is 2.71 and the density of water is 1 g/cc. The shrinkage limit (in %, up to two decimal places) is ______

Key: 19.92

Exp: Given Mass of dry soil = 88g

$$\Rightarrow \gamma_d = \frac{M_d}{V} = \frac{88}{50}\text{g/cm}^3$$

$$\text{also, } \gamma_d = \frac{G}{1+e}\gamma_W \Rightarrow \frac{88}{50} = \frac{G}{1+e}(1)\left[\because \gamma_W = 1\text{g/cm}^3\right]$$

$$\Rightarrow G = \frac{88}{50}(1+e)$$

$$\Rightarrow e = 0.5397$$

$$\Rightarrow \text{shrinkage limit} = \frac{e}{G} = \frac{0.5397}{2.71} = 0.19915$$

$$\text{In\%, } w_s = 19.92\%$$

16. A 10m wide rectangular channel carries a discharge of $20\text{m}^3/\text{s}$ under critical condition. Using $g=9.81\text{m/s}^2$, the specific energy (in m, up to two decimal places) is _______

Key: 1.11

Exp: Specific energy for a rectangular channel

$$= E_c = 1.5y_c$$

$$y_c = \left(q^2/g\right)^{1/3} = \left[\frac{(20/10)^2}{9.81}\right]^{1/3} \left[q = Q/B = 20/10\right]$$

$$= 0.7417\text{m}$$

$$\Rightarrow E_c = 1.5\times 0.7417 = 1.11\text{m}$$

17. The Le Chatelier apparatus is used to determine

(A) Compressive strength of cement (B) fineness of cement
(C) setting time of cement (D) soundness of cement

Key: (D)

Exp: Le-chatelier apparatus is used to determine soundness of cement

18. A city generates 40×10^6 kg of municipal solid waste (MSW) per year, out of which only 10% is recovered / recycled and the rest goes to landfill. The landfill has a single lift of 3m height and is compacted to a density of 550 kg/m^3. If 80% of the landfill is assumed to be MSW, the landfill area (in m^2, up to one decimal place) required would be______

Key: 27272.7

Exp: Total waste (MSW) generated for year $=40\times10^6$ Kg

MSW that goes to landfill $=0.9\times40\times10^6$
$=36\times10^6\,\text{kg}$

Compacted density of waste $=550\,\text{kg/m}^3$

$$\Rightarrow \text{Total valume of waste} = \frac{36\times10^6}{550\,\text{kg/m}^3} = 65454.54\text{m}^3$$

Let A m^2 be landfill area required

$$\Rightarrow \text{MSW land area} = 0.8A = \frac{65454.54}{\text{m}}\text{m}^3 \quad \Rightarrow A = 27272.7\text{m}^2$$

19. In a fillet weld, the direct shear stress and bending tensile stress are 50MPa and 150MPa, respectively. As per IS 800:2007, the equivalent stress (in MPa up to two decimal places) will be ________

Key: 173.205 MPa

Exp: As per Is 800:2007, Equivalent resultant stress

$$=\sqrt{f^2+3.q^2}\le\frac{f_u}{\sqrt{3}.\gamma_m}$$

$$=\sqrt{150^2+3\times50^2}=173.205\ \text{MPa}$$

20. A core cutter of 130mm height has inner an outer diameters of 100 mm and 106mm, respectively. The area ratio of the core cutter (in % up to two decimal places) is _______

Key: 12.26

Exp: Area ratio

$$=A_r=\frac{D_0^2-D_i^2}{D_i^2}\times100\%=\frac{106^2-100^2}{100^2}\times100\%$$

$$\Rightarrow A_r=12.36\%$$

21. Two rectangular under-reinforced concrete beam sections X and Y are similar in all aspects except that the longitudinal compression reinforcement in section Y is 10% more. Which one of the following is the correct statement?
    (A) Section X has less flexural strength and is less ductile than section Y
    (B) Section X has less flexural strength but is more ductile than section Y
    (C) Section X and Y have equal flexural strength but different ductility
    (D) Section X and Y have equal flexural strength and ductility

Key: (A)

Exp: As the compression reinforcement in section Y is more than x so the section is under reinforced section $\Rightarrow$ strength is more, and more ductility

22. For routing of flood in a given channel using the Muskingum method, two of the routing coefficients are estimated as C_0=-0.25 and C_1=0.55. The value of the third coefficient C_2 would be ______

Key: 0.7

Exp: In Muskingum method of flood routing

$$C_0 + C_1 + C_2 = 1$$
$$\Rightarrow -0.25 + 0.55 + C_2 = 1$$
$$\Rightarrow C_2 = 0.7$$

23. The deformation in concrete due to sustained loading is
    (A) creep (B) hydration (C) segregation (D) shrinkage

Key: (A)

Exp: Deformation in concrete due to sustained loading is called creep

24. Bernoulli's equation is applicable for
    (A) viscous and compressible fluid flow
    (B) Inviscid and compressible fluid flow
    (C) Inviscid and incompressible fluid flow
    (D) viscous and incompressible fluid flow.

Key: (C)

Exp: Bernoulli's equation is applicable for ideal fluids i.e for inviscid & incompressible fluid flow

25. Which one of the following matrices is singular?
    (A) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

Key: (C)

Exp: A square matrix 'A' is said to be singular if $|A| = 0$

$$\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 12 - 12 = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \text{is singular matrix.}$$

26. Variation of water depth (y) in a gradually varied open channel flow is given by the first order differential equation

$$\frac{dy}{dx} = \frac{1 - e^{-\frac{10}{3}\ln(y)}}{250 - 45e - ^{-3\ln(y)}}$$

Given initial condition: y(x=0) =0.8m. The depth (in m, up to three decimal places) of flow at a downstream section at x=1m from one calculation step of Single step Euler Method is _______

Key: (0.793)

Exp: From Euler method; we have

$$y_1 = y_0 + hf\left(x_0, y_0\right) \to (1)$$

Given D.E is $\dfrac{dy}{dx} = \dfrac{1 - e^{\frac{-10}{3}\ell n(y)}}{250 - 45e^{-3\ell n(y)}}$ & initial condition $x_0 = 0; y_0 = 0.8$

The above D.E is of the form $\dfrac{dy}{dx} = f(x, y)$, where

$$f(x, y) = \frac{1 - e^{\frac{-10}{3}\ell n(y)}}{250 - 45e^{-3\ell n(y)}} \text{ \& } y \to \text{depth of flow.}$$

$\therefore$ The depth of flow at $x = 1m$ is

$$y(1) = 0.8 + 1.f(0, 0.8)\left[\because \text{form}(1)\ h = 1; x_0 = 0;\ y_0 = 0.8\right]$$

$$\Rightarrow y(1) = 0.8 + \frac{1 - e^{\frac{-10}{3}\ell n(0.8)}}{250 - 45e^{-3\ell n(0.8)}}$$

$$= 0.8 + \frac{-1.103940125}{162.1093751} \cong 0.793$$

27. An aircraft approaches the threshold of a runway strip at a speed of 200km/h. The pilot decelerates the aircraft at a rate of 1.697m/s^2 and takes 18s to exit the runway strip. If the deceleration after exiting the runway is 1 m/s^2, then the distance (in m, up to one decimal place) of the gate position from the location of exit on the runway is _______

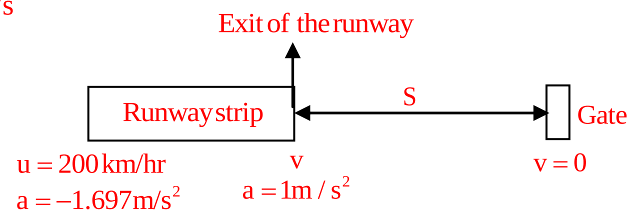
Key: 313

Exp: Let speed of aircraft at the exit of runway be V m/s

$$\Rightarrow V = 200 \times \left(\frac{5}{18}\right) - 1.697 \times 18 = 25.02\,m/s$$

velocity at gate position = 0

$$\Rightarrow 0^2 = (25.02)^2 - 2 \times 1 \times S \Rightarrow S = 313m$$



28. A water sample analysis data is given below

Ion	Concentration, mg/L	Atomic Weight
Ca^{2+}	60	40
Mg^{2+}	30	24.31
HCO_3^-	400	61

The carbonate hardness (expressed as mg/L of $CaCO_3$, up to one decimal place) for the water sample is _______.

Key: 273.406

Exp: Total alkalinity $= [\text{conectration of } HCO_3] \times \frac{50}{61}$

$$= 400 \times \frac{50}{61} = 327.86\ mg/L \text{ as ca } CO_3$$

$$\text{Total hardness} = [\text{conc.of } Mg^{2+}] \times \frac{50}{\left(\frac{24.31}{2}\right)} + [\text{con c .of } ca^{2+}] \times \frac{50}{\left(\frac{40}{2}\right)}$$

$$= 30 \times \frac{50 \times 2}{24.31} + 60 \times \frac{2 \times 50}{40} = 123.406 + 150 = 273.406\ mg/1 \text{as } caco_3$$

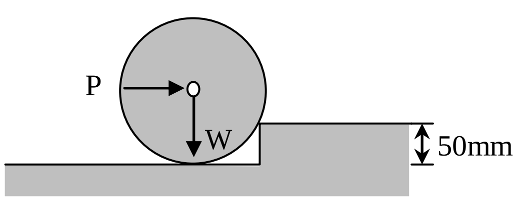
$TA > TH \Rightarrow CH = TH$

$NCH = 0$

$CH = 273.406\ mg/1 \text{ as } caco_3$

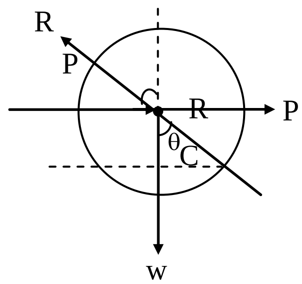
29. A cylinder of radius 250mm and weight, W=10kN is rolled up an obstacle of height 50mm by applying a horizontal force P at its centre as shown in the figure.
All interfaces are assumed frictionless. The minimum value of P is

(A) 4.5kN
(B) 5.0kN
(C) 6.0kN
(D) 7.5kN

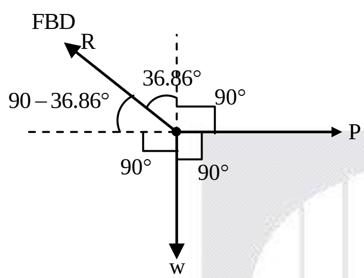


Key: (D)

Exp:



$$\theta = \cos^{-1}\left(\frac{200}{250}\right) = 36.86°$$



By Lamis theorem

$$\frac{P}{\sin(90+90-36.86)} = \frac{w}{\sin(90+36.80)} = \frac{R}{\text{Sin}90}$$

$$P = \frac{w.\sin(143.14)}{\sin(126.86)}(w = 10\text{KN}) = 7.49\text{KN}$$

30. An RCC beam of rectangular cross section has factored shear of 200kN at its critical section. Its width b is 250 mm and effective depth d is 350mm. Assume design shear strength τ_c of concrete as 0.62 N/mm^2 and maximum allowable shear stress $\tau_{c.max}$ in concrete as 2.8 N/mm^2. If two legged 10mm diameter vertical stirrups of Fe250 grade steel are used, then the required spacing (in cm, up to one decimal place) as per limit state method will be_______

Key: 8.2

Exp: $\tau_c = 0.62\,\text{N/mm}^2$

$\tau_{c},_{max} = 2.8\,\text{N/mm}^2$

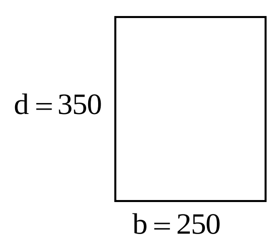
$\tau_v = \frac{V_u}{bd} = \frac{200\times103}{250\times350} = 2.286\,\text{N/mm}^2$

$\tau_{c1}\,\text{max} = 2.8\,\text{N/mm}^2$

$\tau_v < \tau_{c_1}\,\text{max} \rightarrow \text{safe}$

As per 456 – 2000

For vertical stirrups to be provided



$$V_{us} = 0.87\,f_y.A_{s.}\frac{d}{s_v}$$

$$(\tau_v - \tau_c)bd = 0.87f_y.A_{sv}\frac{d}{s_v}$$

$$(2.28 - 0.62)250 \times 350 = 0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 10^2 \times \frac{350}{s_v}$$

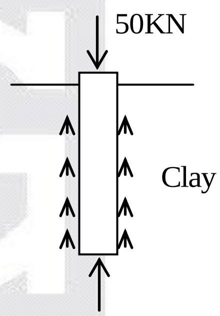
$$s_v = 82mm = 8.2cm$$

31. A 0.5m×0.5m square concrete pile is to be driven in a homogeneous clayey soil having undrained shear strength, c_u =50kPa and unit weight, $\gamma = 18.0kN/m^3$, The design capacity of the pile is 500kN. The adhesion factor α is given as 0.75. The length of the pile required for the above design load with a factor of 2.0 is

(A) 5.2m (B) 5.8m (C) 11.8m (D) 12.5m

Key: (C)

Exp: $Q_u = Q_{pu} + Q_{pt}$



$$2 \times 500KN = A_b.f_b + A_s f_s$$

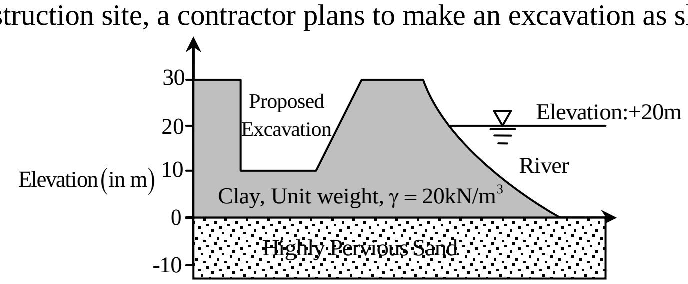
$$100 = (0.5 \times 0.5) \times (9c) + (\propto c)A_{surface}$$

$$1000 = 0.5 \times 0.5 \times 9 \times 50 + 0.75 \times 50 \times 4 \times 0.5 \times L$$

$$1000 = 112.5 + 75L$$

$$L = 11.833m$$

32. At a construction site, a contractor plans to make an excavation as shown in the figure.



The water level in the adjacent river is at an elevation of +20.0m. Unit weight of water is 10kN/m³. The factor of safety (up to two decimal places) against sand boiling for the proposed excavation is ________

Key: (1)

Exp: Uplift pressure due to pore water pressure $= 20 \times \gamma_w = 200\,kN/m^3$

Total downward pressure at interface of sand and clay after excavation

$$= 10 \times \gamma = 10 \times 20 = 200\,kN/m^3$$

$$\text{Factor of safety} = \frac{\text{Total downward pressure}}{\text{uplift pressure}} = \frac{200}{200} = 1$$

33. A conventional drained triaxial compression test was conducted on a normally consolidated clay sample under an effective confining pressure of 200kPa. The deviator stress at failure was found to be 400kPa. An identical specimen of the same clay sample is isotropically consolidated to a confining pressure of 200kPa and subjected to standard undrained triaxial compression test. If the deviator stress at failure is 150kPa, the pore pressure developed (in kPa, up to one decimal place) is ___________

Key: 125

Exp: For drained

$\sigma_c = 200\text{kPa},\ \sigma_d = 400\text{kPa}$

$\text{Pore pressure} = 0$

$C = 0$

$\sigma_1 = \sigma_c + \sigma_d = 200 + 400 = 600\text{KPa}$

$\sigma_3 = \sigma_c = 200\text{KPa}$

$$\sigma_1 = \sigma_3 \tan^2\left(45\frac{\phi}{2}\right)$$

$$600 = 200\tan^2\left(45 + \frac{\phi}{2}\right)$$

$\phi = 30$

For un drained

$\sigma_c = 200\text{kPa},\ \sigma_d = 150\text{kPa}$

$\bar{\sigma}_1 = \bar{\sigma}_c + \bar{\sigma}_d = 200 + 150 = 350$

$\bar{\sigma}_1 = 350 - u$

$$\sigma_3 = \sigma_c - u = 200 - u \Rightarrow \bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$(350 - u) = (200 - u)\tan^2\left(457\frac{30}{2}\right)$$

$350 - u = 600 - 3u$

$3u - u = 600 - 350$

$$u = \frac{250}{2} = 125\text{KPa}$$

34. The solution (up to three decimal places) at x=1 of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ subject to boundary conditions $y(0) = 1$ and $\frac{dy}{dx}(0) = -1$ is _______

Key: 0.368

Exp: Given D.E is

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$\Rightarrow \left(D^2+2D+1\right)y=0 \Rightarrow (D+1)^2 y=0 \rightarrow (1)$

$\therefore$ The auxilary equation is

$$(m+1)^2=0$$

$\Rightarrow m=-1,-1$ [Real & Repeated]

The complementary function (C.F)

$y_c = e^{-x}[c_1+c_2x]$

&

$\therefore y_p = 0$ [particular integral]

$\therefore$ The complete solution of (1) is

$y=e^{-x}[c_1+c_2x] \rightarrow (2)$

Given that $y(0)=1$

from $(1) \Rightarrow 1=c_1+0 \Rightarrow c_1=1$

$\dfrac{dy}{dx}=-e^{-x}[c_1+c_2x]+e^{-x}[c_2]$

Given $\dfrac{dy}{dx}=-1$ at $x=0$

$-1=-[1+0]+1[c_2] \Rightarrow c_2=0$

$\therefore y=e^{-x}[1+0]=e^{-x} \Rightarrow y(1)=e^{-1} \simeq 0.368$

II method:

Given D.E is $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=0$

$\Rightarrow y''+2y'+y=0$

using L.T on both sides;

$L[y'']+2L[y']+L[y]=0$

$\Rightarrow s^2L[y(x)]-sy(0)-y'(0)+2[SL[y(x)]-y(0)]+L[y]=0$

$\Rightarrow \left(s^2+2s+1\right)L[y(x)]-s(1)-(-1)-2(1)=0$

$[\because y(0)=1 \,\&\, y'(0)=-1]$

$\Rightarrow \left(s^2+2s+1\right)L[y(x)]=1+s$

$\Rightarrow L[y(x)]=\dfrac{1+s}{s^2+2s+1}=\dfrac{1+s}{(1+s)^2}=\dfrac{1}{1+s}$

applying inverse L.T;

$y(x)=L^{-1}\left[\dfrac{1}{s+1}\right]=e^{-x} \Rightarrow y(1)=e^{-1} \simeq 0.368.$

35. Rainfall depth over a watershed is monitored through six number of well distributed rain gauges. Gauged data are given below.

Rain Gauge Number	1	2	3	4	5	6
Rainfall Depth(mm)	470	465	435	525	480	510
Area of Thiessen Polygon $\left(\times 10^4 m^2\right)$	95	100	98	80	85	92

The Thiessen mean value (in mm, up to one decimal place) of the rainfall is _________

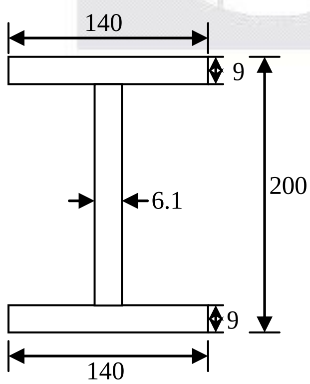
Key: 479.1

Exp: Thiessen's mean value $= \dfrac{\sum_{i=1}^{6} P_i A_i}{\Sigma A_i}$

$$= \frac{\begin{bmatrix}(470\times95)+(465\times100)+(435\times98)+(525\times80)\\ +(480\times85)+(510\times92)\end{bmatrix}\times10^4}{(95+100+98+80+85+92)\times10^4}$$

$$= \frac{263,500\times10^4}{550\times10^4} = 479.1\text{mm}$$

36. The dimensions of a symmetrical welded I-section are shown in the figure.



(All dimensions are in mm)

The plastic section modulus about the weaker axis (in cm^3, up to one decimal place) is ___________.

Key: 89.9

Exp: weaker axis is y-axis, because $I_{yy} < I_{xx}$

$$Z_p = \frac{A}{Z}\left(\bar{y}_1 + \bar{y}_2\right)$$

$$A = (140\times9)\times2 + 6.1\times(200-2\times9)$$

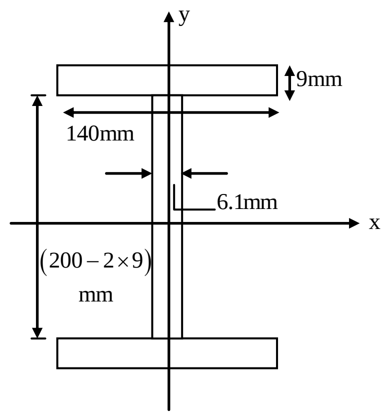
$$= 3630.2\text{mm}^2$$

Due to symetry $\bar{y}_1 = \bar{y}_2 =$ Distance of centroid of compression/ tension region form eaqual area axis equal area axis is vertical y-axis passes form centroid of I section as shown.

$$\Rightarrow \bar{y}_1 = \frac{(70\times9)\times\left(\frac{70}{2}\right)+\frac{6.1}{2}\times(200-2\times9)\times\frac{6.1}{4}+(70\times9)\times\left(\frac{70}{2}\right)}{(70\times9)+\frac{6.1}{2}\times(200-18)+(70\times9)}$$

$$= \frac{44946.52}{1815.1}$$

$$= 24.76 \text{ mm}$$



$$\Rightarrow Z_p = \frac{3630.2}{2}[24.76+24.76]$$

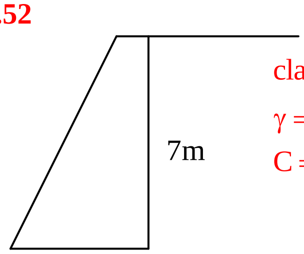
$$= 89883.75\text{mm}^3$$

$$z_p \simeq 89.9 \text{ cm}^3$$

37. A rigid smooth retaining wall of height 7m with vertical backface retains saturated clay as backfill. The saturated unit weight and undrained cohesion of the backfill are 17.2kN/m^3 and 20kPa, respectively. The difference in the active lateral forces on the wall (in kN per meter length of wall, up to two decimal places), before and after the occurrence of tension cracks is _____.

Key: 46.52

Exp:

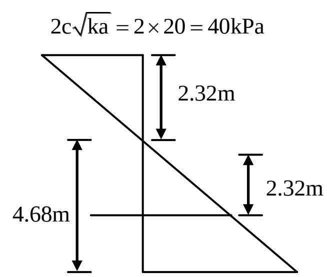


$\text{clay} \Rightarrow \phi = 0$

$\gamma = 17.2 \text{KN/m}^3$

$C = 20\text{KPa}$

$$k_a = \frac{1-\sin 0}{1+\sin 0} = 1$$



Before occurrence of tension crack

$$z_c = \frac{2C}{\gamma\sqrt{k_a}} = \frac{2\times20}{17.2\sqrt{1}} = 2.32\text{m}$$

Before formation of tension crack

$$k_a\gamma H - 2c\sqrt{k_a} = 1\times17.2\times7-2\times20 = 80.4\text{kPa}$$

$$F_a = \frac{1}{2}k_a\gamma H^2 - 2c\sqrt{k_a}H = \frac{1}{2}\times1\times17.2\times7^2 - 2\times20\sqrt{1}\times7 = 141.4\text{kN/m}$$

After formation of tension crack

$$F_a = k_a\gamma\frac{H^2}{2} - 2c\sqrt{k_a}H + \frac{2c^2}{\gamma} = 1\times 17.2\times\frac{7^2}{2} - 2\times 20\times\sqrt{1}\times 7 + 2\times\frac{20^2}{17.2}$$
$$= 421.4 - 280 + 46.52$$
$$= 187.92\,\text{kN/m}$$

difference $= 187.92 - 141.4 = 46.52\,\text{kN/m}$

38. A waste activated sludge (WAS) is to be blended with green waste (GW). The carbon (C) and nitrogen (N) contents. per kg of WAS and GW, on dry basis are given in the table.

Parameter	WAS	GW
Carbon (g)	54	360
Nitrogen (g)	10	6

The ratio of WAS to GW required (up to two decimal places) to achieve a blended C:N ratio of 20:1 on dry basis is ________

Key: 1.643

Exp: Let $\frac{WAS}{GW}$ be x

Assume 1kg of GW $\Rightarrow$ WAS = x kg

$\therefore$ Total carbon (c) in WAS & GW $= 360 + 54x$ grams

Similarly Total nitrogen (N) in WAS & 4w $= 6 + 10x$ gm

$$\therefore \frac{C}{N} = \frac{20}{1}$$
$$\Rightarrow \frac{360+54x}{6+10x} = \frac{20}{1}$$
$$\Rightarrow 360 + 54x = 120 + 200x$$
$$\Rightarrow x = \frac{WAS}{GW} = 1.643$$

39. The infiltration rate f in a basin under ponding condition is given by $f=30+10e^{-2t}$, where, f is in mm/h and t is time in hour. Total depth of infiltration (in mm, up to one decimal place) during the last 20 minutes of a storm of 30 minutes duration is _______.

Key: 11.74

Exp: Total infiltration in last 20 minutes of 30 minutes storm

i.e form $\frac{10}{60}$ hr to $\frac{30}{60}$ hr

1.667hrs to 0.5 hr

$$\int_{1.667}^{0.5}\left(30+10.e^{-2t}\right)dt = 30t + 10\frac{e^{-2t}}{-2}\Big|_{1.667}^{0.5} = 11.74\text{mm}$$

40. In laboratory, a flow experiment is performed over a hydraulic structure. The measured values of discharge and velocity are $0.05\text{m}^3/\text{s}$ and 0.25m/s, respectively. If the full scale structure (30 times bigger) is subjected to a discharge of $270\text{m}^3/\text{s}$, then the time scale (model to full scale) value (up to two decimal places) is ______

Key: 0.2

Exp: Given model values as

$$Q_m = 0.05\,\text{m}^3/\text{s};\ V_m = 0.25\,\text{m/s}$$

$$\Rightarrow Q_m = V_m x (L_m)^2 \Rightarrow L_m^2 = \frac{0.05}{0.25} = 0.2\text{m}^2$$

$$\text{Given } \frac{L_m}{L_p} = \frac{1}{30}\left[\text{Prototype is full scale strcture having } L_p = 30L_m\right]$$

$$\Rightarrow L_p^2 = 30^2 \times 0.2\left[\text{Suffix 'p' denetes prototype values}\right] = 180\text{m}^2$$

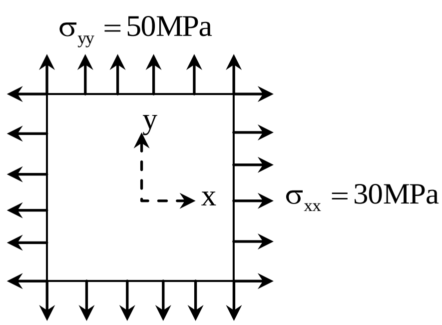
$$\Rightarrow V_p = \frac{Q_p}{Lp^2} = \frac{270}{180\text{m}^2}\text{m}^3/\text{s} = 15.\text{m/s}$$

$$\Rightarrow \text{Time scale} = T_\gamma = \frac{T_m}{T_p} = \frac{L_m/V_m}{L_p/V_p} = \frac{L_m V_p}{L_p V_m}$$

$$\Rightarrow \text{Time scale (model to full scale)}$$

$$= \left(\frac{1}{30}\right) \times \left(\frac{1.5}{0.25}\right) = 0.2$$

41. A plate in equilibrium is subjected to uniform stresses along its edges with magnitude $\sigma_{xx} = 30\text{MPa}$ and $\sigma_{yy} = 50\text{MPa}$ as shown in the figure.



The Young's modulus of the material is $2\times10^{11}\text{N/m}^2$ and the Poisson's ratio is 0.3 if σ_{zz} is negligibly small and assumed to be zero. Then the strain ε_{zz} is

(A) -120×10^{-6} (B) -60×10^{-6} (C) 0.0 (D) 120×10^{-6}

Key: (A)

Exp:
$$\varepsilon_z = \frac{\sigma_z - \mu\sigma_x - \mu\sigma_y}{E}$$
$$= \frac{(0 - 0.3\times30 - 0.3\times50)\,N/mm^2}{2\times10^{11}\,N/m^2} = -\frac{12\times10^6}{10^{11}} = -120\times10^{-6}$$

42. An RCC short column (with lateral ties) of rectangular cross section of 250mm×300mm is reinforced with four numbers of 16mm diameter longitudinal bars. The grades of steel and concrete are Fe415 and M20, respectively. Neglect eccentricity effect. Considering limit state of collapse in compression (IS 456:2000), the axial load carrying capacity of the column (in kN, up to one decimal place), is ______________.

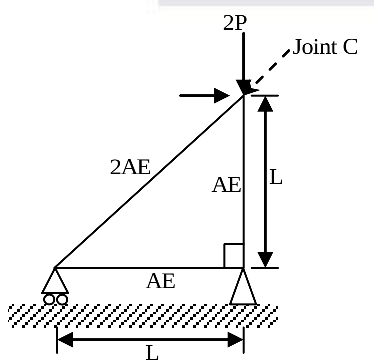
Key: 917.96

Exp: Neglecting electricity effect, column can be assumed as concentrically loaded

Accordinge to IS 456:200; axial load carrying capacity for concentrically loaded column is

$$P_u = 0.45\,f_{ck}Ag + (0.75f_y - 0.45f_{ck})A_{sc}$$
$$= [0.45\times20\times250\times300] + [0.75\times415 - 0.45\times20]\times4\times\frac{\pi}{4}\times16^2$$
$$P_u = 917.96\text{ KN}$$

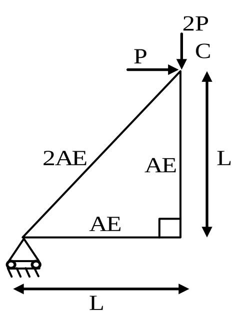
43. Consider the deformable pin-jointed truss with loading, geometry and section properties as shown in the figure.



Given that E=2×10^{11}N/m^2, A=10mm^2, L =1 m and P =1 kN. The horizontal displacement of joint C (in mm, up to one decimal place) is ___________

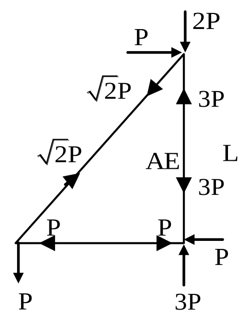
Key: 2.707

Exp:

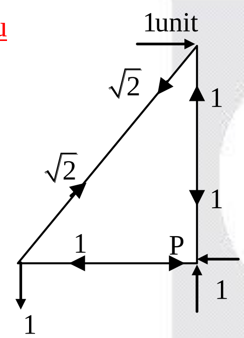


$\Delta_c]\text{horizontal} = \Sigma \frac{P_i u_i L}{AE}$

Calculation of P



Calculation of u



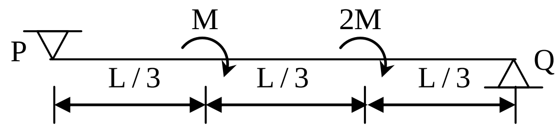
Applying 1 unit load in horizontal direction at C.

$$\Rightarrow \Delta_C = \frac{(\sqrt{2}P)(\sqrt{2})\sqrt{2}L}{2AE} + \frac{(3P)(1)\times L}{AE} + \frac{(P)(1)(L)}{AE}$$

$$\Rightarrow \Delta_C]_{\text{horizontal}} = \frac{PL}{AE}(4+\sqrt{2}) = \frac{(1000)(1)(4+1.414)\times 1000}{10\times 2\times 10^{11}/10^6}$$

$$\Delta_C]_{\text{horizontal}} = 2.707\text{ mm}$$

44. The figure shows a simply supported beam PQ of uniform flexural rigidity EI carrying two moments M and 2M.

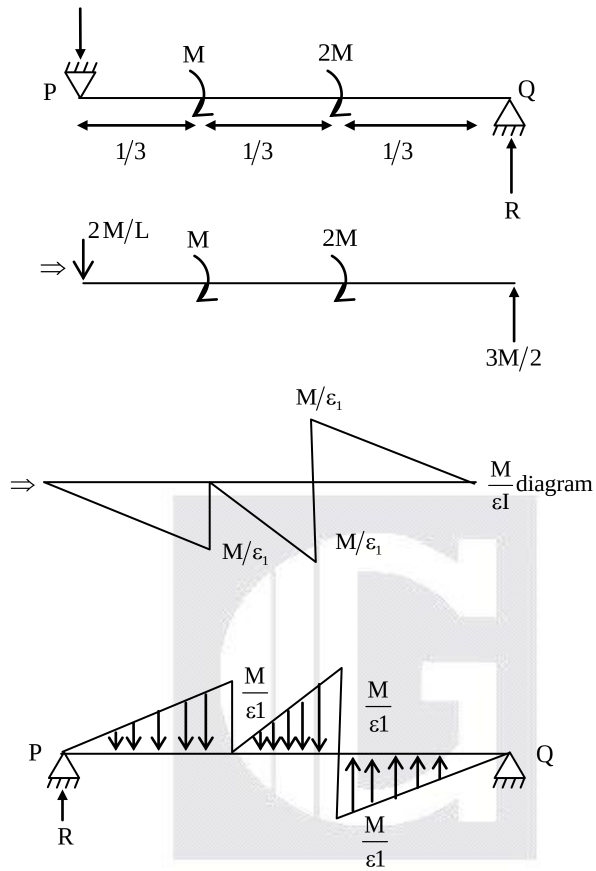


The slope at P will be

(A) 0 (B) ML/(9EI) (C) ML/(6EI) (D) ML/(3EI)

Key: (C)

Exp:



$\Sigma Q = 0 \Rightarrow R \times L - M - 2M = 0$

$\Rightarrow R = 3M/L$

Slope at P = S.F at P of conjugate beam = R

$\Rightarrow \Sigma M_Q = 0$

$$\Rightarrow R \times (L) - \left(\frac{M}{EI}\right) \times \frac{1}{2} \times L/3 \times \left[\frac{2L}{3} + \frac{L}{9}\right] + \left(\frac{ML}{EI}\right) \times \frac{1}{2} \times \frac{L}{3}[L/3 + L/9]$$

$$+\left(\frac{ML}{EI}\right) \times \frac{1}{2} \times \frac{L}{3}[2L/9]$$

$$\Rightarrow RL - \frac{7ML}{54EI} - \frac{4ML}{54EI} + \frac{2ML}{54EI} = 0$$

$$\Rightarrow R = \frac{ML}{6EI} = \text{slope at P}$$

45. The solution at x=1, t=1 of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of $u(0) = 3x$ and $\frac{\partial u}{\partial t}(0) = 3$ is ________

(A) 1 (B) 2 (C) 4 (D) 6

Key: (D)

Exp: By D'Alembert's formula

Solution to IVP for wave equation

$u_{tt} = C^2 U_{xx}$

$u(x,0) = f(x)$

$u_t(x,0) = g(x)$

is $u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds - (1)$

Given $f(x) = 3x$

$g(x) = 3 \quad \& \quad c^2 = \frac{1}{25}$

from (1) $u(x,t) = \frac{1}{2}\left[3\left(x+\frac{t}{5}\right) + 3\left(x-\frac{t}{5}\right)\right] + \frac{1}{2\left(\frac{1}{5}\right)}\int_{x-\frac{t}{5}}^{x+\frac{t}{5}} 3\,ds$

$= 3x + 3t$

At $x = 1, t = 1; u(x,t) = 6.$

46. A square area (on the surface of the earth) with side 100m and uniform height, appears as $1cm^2$ on a vertical aerial photograph. The topographic map shows that a contour of 650m passes through the area. IF focal length of the camera lens is 150mm, the height from which the aerial photograph was taken, is

(A) 800m (B) 1500m (C) 2150m (D) 3150m

Key: C

Exp: Elevation of square area from datum = control value =650 m

Scale of photograph $= \frac{1cm}{100m} = \frac{1}{10000}$

$\Rightarrow S = \frac{f}{H-h}$

$\Rightarrow \frac{1}{10,000} = \frac{150\,mm}{H - 650m}$

$\Rightarrow H - 650 = 0.150 \times 10,000$

$\Rightarrow H - 650 = 0.150 \times 10,000$

$\Rightarrow H = 1500 + 650$

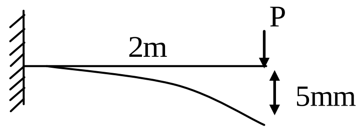
$\Rightarrow H = 2,150m$

47. A cantilever beam of length 2m with a square section of side length 0.1 m is loaded vertically at the free end. The vertical displacement at the free end is 5mm. The beam is made of steel with Young's modulus of 2.0×10^{11} N/m^2. The maximum bending stress at the fixed end of the cantilever is

(A) 20.0MPa (B) 37.5MPa (C) 60.0MPa (D) 75.0MPa

Key: (B)

Exp: Given Δmax at free end = 5mm



Also, Δ_{max} at free end $=\dfrac{Pl^3}{3.EI}$

$$\Rightarrow 0.005=\frac{P(2)^3}{3.EI}\Rightarrow P=1.875\times10^{-3}EI\left[I=\frac{(0.1)^4}{12}\right]$$

$$\text{Max bending stress at fixed end}=\frac{M}{Z}=\frac{6M}{b.d^2}\quad\left[\begin{array}{l}b=d=0.1m\\ M=P\times2=2P\end{array}\right]$$

$$=\frac{6\times2P}{(0.1)^3}$$

$$=\frac{12}{0.001}\times1.875\times10^{-3}\times2\times10^{11}\times\frac{(0.1)^4}{12}=37.5\,MPa$$

48. The value of the integral $\int_0^{\pi} x\cos^2 x dx$ is

(A) $\pi^2/8$ (B) $\pi^2/4$ (C) $\pi^2/2$ (D) π^2

Exp:

we know that

$$\int_a^b x f(x).dx=\frac{b-a}{2}\int_a^b f(x).dx \text{ if } f(a+b-x)=f(x)$$

Let $f(x)=\cos^2 x$

$$\Rightarrow f(\pi-x)=\cos^2(\pi-x)=\cos^2 x=f(x)$$

$$\therefore \int_0^{\pi} x\cos^2 x\,dx=\frac{\pi}{2}\int_0^{\pi}\cos^2 x\,dx$$

$$=\frac{\pi}{2}\int_0^{\pi}\left[\frac{1+\cos2x}{2}\right]dx\qquad\left[\begin{array}{l}\therefore \cos2\theta=2\cos^2\theta-1\\ \cos^2\theta=\dfrac{1+\cos2\theta}{2}\end{array}\right]$$

$$=\frac{\pi}{4}\left[x+\frac{\sin2x}{2}\right]_0^{\pi}$$

$$=\frac{\pi}{4}\{(\pi+0)-(0+0)\}=\frac{\pi}{4}(\pi)=\frac{\pi^2}{4}$$

$$\int_0^{\pi} x\cos^2 x\,dx = \frac{\pi^2}{4}$$

Method II

$$\int_0^{\pi} x\cos^2 x\,dx = \int_0^{\pi} x\left[\frac{1+\cos 2x}{2}\right]dx \qquad \left[\begin{array}{l}\therefore \cos 2\theta = 2\cos^2\theta - 1\\ \cos^2\theta = \dfrac{1+\cos 2\theta}{2}\end{array}\right]$$

$$= \int_0^{\pi}\left[\frac{x}{2}+\frac{x\cos 2x}{2}\right]dx$$

$$= \frac{1}{2}\left\{\frac{x^2}{2}+x\frac{\sin 2x}{2}+\frac{1}{4}\cos 2x\right\}_0^{\pi}$$

$$= \frac{1}{2}\left\{\left[\frac{\pi^2}{2}+0+\frac{1}{4}\right]-\left[0+0+\frac{1}{4}\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{\pi^2}{2}+\frac{1}{4}-\frac{1}{4}\right\} = \frac{\pi^2}{4}$$

49. The void ratio of a soil is 0.55at an effective normal stress of 140kPa. The compression index of the soil is 0.25. In order to reduce the void ratio to 0.4, an increase in the magnitude of effective normal stress (in kPa, up to one decimal place) should be ________

Key: 417.37

Exp: Given

$e_1 = 0.55$; $\sigma_1 = 140$ kPa

$e_2 = 0.4$; $\sigma_2 = ?$

$$C_c = \frac{\Delta e}{\log\left(\frac{\sigma_2}{\sigma_1}\right)} \Rightarrow 0.25 = \frac{0.55-0.4}{\log\left(\frac{\sigma_2}{140}\right)}$$

$$\Rightarrow \sigma_2 = 557.35\,\text{kN}$$

Increase in magnitude of stress = $\Delta\sigma = \sigma_2 - \sigma_1 = 557.35 - 140$

$$\Delta\sigma = 417.35\,\text{kPa}$$

50. A closed tank contains 0.5m thick layer of mercury (specific gravity=13.6) at the bottom. A 2.0m thick layer of water lies above the mercury layer. A 3.0 m thick layer of oil (specific gravity=0.6) lies above the water layer. The space above the oil layer contains air under pressure. The gauge pressure at the bottom of the tank is 196.2 kN/m^2. The density of water is 1000kg/m^3 and the acceleration due to gravity is 9.81m/s^2. The value of pressure in the air space is

(A) 92.214kN/m^2 (B) 95.644kN/m^2 (C) 98.922kN/m^2 (D) 99.321kN/m^2

Key: (A)

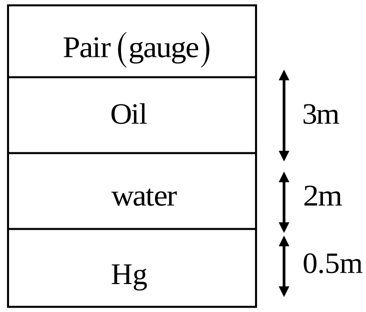
Exp: Pressure at bottom (guage)

$$= P_{air(gauge)} + 0.6\times3\times\gamma_w + 1\times2\times\gamma_w + 13.6\times0.5\times\gamma_w$$

$$= 10.6\gamma_w + P_{air(guage)}$$

$$\Rightarrow 196.2 = 10.6\times1000\times9.81\times10^{-3} + P_{air(guage)}$$

$$\Rightarrow P_{air(guage)} = 92.214\,kN/m^2$$



51. The following details refer to a closed traverse:

Line	Consecutive Coordinate			
	Northing (m)	Southing (m)	Easting (m)	Westing (m)
PQ	------	437	173	-----
QR	101	-----	558	-----
RS	419	-----	-----	96
SP	------	83	-----	634

The length and direction (Whole circle bearing) of closure, respectively are

(A) 1 m and 90° (B) 2m and 90° (C) 1 m and 270° (D) 2m and 270°

Key: (A)

Exp: For closed traverse

$$\text{Latitude}(\Sigma L) = \Sigma N - \Sigma S = (101+419)-(437+83) = 520-520 = 0$$

$$\text{Departure}(\Sigma D) = \Sigma E - \Sigma W = (173+558)-(96+634) = 731-730 = 1$$

$$L = \sqrt{(\Sigma L)^2 (\Sigma D)^2} = \sqrt{0^2+1^2} = 1$$

$$\theta = \tan^{-1}\left(\frac{\Sigma D}{\Sigma L}\right)$$

$$\theta = 90° \text{ or } 270°$$

52. Given the following data: design life n=15 years, lane distribution factor D=0.75. annual rate of growth of commercial vehicles r=6%. Vehicle damage factor F=4 and initial traffic in the year of completion of construction =3000 commercial Vehicles Per Day (CVPD). As per IRC:37-2012, the design traffic in terms of cumulative number of standard axles (in million standard axles, up to two decimal places) is ______.

Key: 76.46

Exp: $\text{C.M.S.A} = \left[\dfrac{365A\left((1+r)^n - 1\right)}{r} \times \text{LDF} \times \text{VDF}\right] \times 10^{-6}$

$\text{LDF} = \text{Lane distribution factor} = 0.75$

$\text{VDF} = \text{Vehicle Damage factor} = 4$

A=Initial traffic in the year of completion of construction=3000

n=15 year

r=6%

$$\Rightarrow \text{C.M.S.A} = \frac{365\left((1+0.06)^{15} - 1\right)}{0.06} \times 3000 \times 4 \times 0.75 \times 10^{-6}$$
$$= 76.4615\,\text{MSA}$$

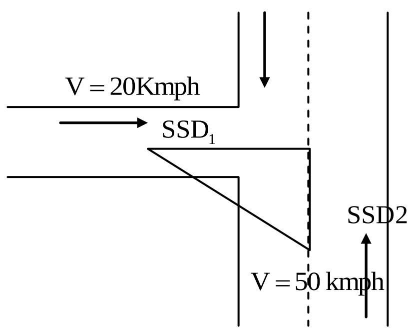
53. A priority intersection has a single lane one way traffic road crossing an undivided two lane two way traffic road. The traffic stream speed on the single lane road is 20kmph and speed on the two lane road is 50kmph. The perception-reaction time is 2.5s, coefficient of longitudinal friction is 0.38 and acceleration due to gravity is 9.81m/s^2. A clear sight triangle has to be ensured at this intersection. The minimum lengths of the sides of the sight triangle along the two lane road and the single lane road, respectively will be

(A) 50m and 20m (B) 61m and 18m

(C) 111m and 15m (D) 122m and 36m

Key: (B)

Exp:



$$\text{SSD}_1 = v.t_r + \frac{v^2}{2g\mu} = \left(20 \times \frac{5}{18}\right) \times 2.5 + \frac{\left(20 \times \frac{5}{18}\right)^2}{2 \times 9.81 \times 0.38} = 13.89 + 4.139 = 18\text{m}$$

$$\text{SSD}_2 = v.t_r + \frac{v^2}{2g\mu} = \left(50 \times \frac{5}{18}\right) \times 2.5 + \frac{\left(50 \times \frac{50}{18}\right)^2}{2 \times 9.81 \times 0.38}$$
$$= 34.72 + 25.87 = 60.59 \cong 61\text{m}$$

54. The ultimate BOD (L_0) of a wastewater sample is estimated as 87% of COD. The COD of this wastewater is 300mg/L. Considering first order BOD reaction rate constant k (use natural Log) = 0.23 per day and temperature coefficient $\theta = 1.047$, the BOD value (in mg/L, up to one decimal place) after three days of incubation at 27°C for this wastewater will be ________

Key: 160.22

Exp: k =0.23/day(natural log)-at temp=20^0C

$$k_{(T^0C)} = k_{(20^0C)}[1.047]^{T-20^0C}$$

$$\Rightarrow k_{(27^0C)} = 0.23[1.047]^{27-20} = 0.3172 / \text{day}$$

$$\text{ultimate BOD} = 0.87 \times 300 = 261 \text{mg / L}$$

$$\text{BOD for 3 days} = 261\left(1 - e^{-k \times 3}\right) = 261\left(1 - e^{-0.3172 \times 3}\right) = 160.22 \text{mg / L}$$

55. A rapid sand filter comprising a number of filter beds is required to produce 99MLD of potable water. Consider water loss during backwashing as 5%, rate of filtration as 6.0m/h and length to width ratio of filter bed as 1.35. The width of each filter bed is to be kept equal to 5.2m. One additional filter bed is to be provided to take care of break-down, repair and maintenance. The total number of filter beds required will be

(A) 19 (B) 20 (C) 21 (D) 22

Key: (C)

Exp: Total Volume of water required to be filtered

Including backwashing water $= 99 + 0.05 \times 99 = 103.95 \text{MLD}$

Area of each filter $= L \times B = (1.35 \times 5.2) \times 5.2 = 36.504 \text{m}^2$

$$\Rightarrow \text{Volume of water filtered in 1hour} = 6 \times 36.504$$

$$= 219.024 \text{ m}^3$$

$$\Rightarrow \text{Volume of water filtered in 1day} = \frac{219.024 \times 24 \times 10^3}{10^6} \text{MLD}$$

$$\Rightarrow \text{No.of filter beds required} = \frac{103.95}{5.256} = 19.77$$

Total no.of filter bed required=19.77+1

=20.77

$\simeq 21$