GATE SOLVED PAPER - CE

2014-2

General Aptitude

Q. 1 - Q. 5 Carry one mark each.

Q. 1 Choose the most appropriate word fro the options given below to complete the following sentence.
A person suffering from Alzheimer's disease short-term memory loss.

(A) experienced
(B) has experienced
(C) is experiencing
(D) experiences

Q. 2 Choose the most appropriate word from the options given below to complete the following sentence.
_____________ is the key to their happiness; they are satisfied with what they have.

(A) Contentment
(B) Ambition
(C) Perseverance
(D) Hunger

Q. 3 Which of the following options is the closest in meaning to the sentence below?
"As a woman, I have no country."

(A) Women have no country.
(B) Women are not citizens of any country.
(C) Women's solidarity knows no national boundaries.
(D) Women of all countries have equal legal rights.

Q. 4 In any given year, the probability of an earthquake greater than Magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrences of such earthquakes is ____ years.

Q. 5 The population of a new city is 5 million and is growing at 20% annually. How many years would it take to double at this growth rate?

(A) 3-4 years
(B) 4-5 years
(C) 5-6 years
(D) 6-7 years

Q. 6 - Q. 10 Carry two marks each.

Q. 6 In a group of four children, Som is younger to Riaz. Shiv is elder to Ansu. Ansu is youngest in the group. Which of the following statements is/are required to find the eldest child in the group? Statements

1. Shiv is younger to Riaz.
2. Shiv is elder to Som.

(A) Statement 1by itself determines the eldest child.

(B) Statement 2 by itself determines the eldest child.

(C) Statements 1 and 2 are both required to determine the eldest child.

(D) Statements 1 and 2 are not sufficient to determine the eldest child.

Q. 7 Moving into a world of big data will require us to change our thinking about the merits of exactitude. To apply the conventional mindset of measurement to the digital, connected world of the twenty-first century is to miss a crucial point. As mentioned earlier, the obsession with exactness is an artefact of the information-deprived analog era. When data was sparse, every data point was critical, and thus great care was taken to avoid letting any point bias the analysis.
From "BIG DATA" Viktor Mayer-Schonberger and Kenneth Cukier
The main point of the paragraph is:

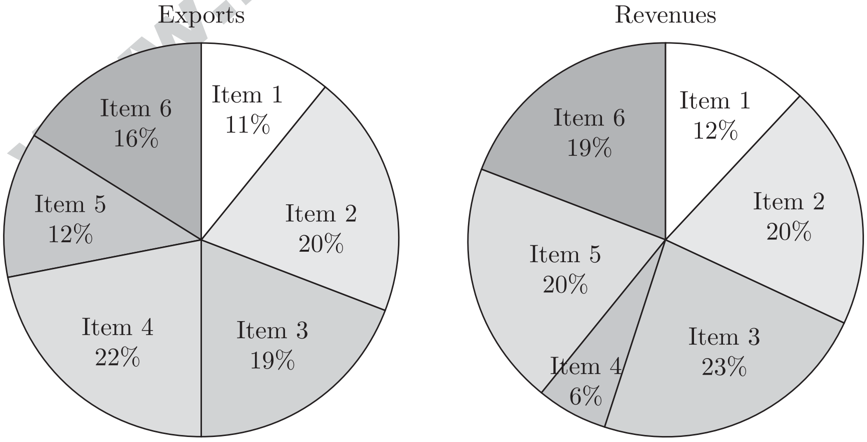
(A) The twenty-first century is a digital world

(B) Big data is obsessed with exactness

(C) Exactitude is not critical in dealing with big data

(D) Sparse data leads to a bias in the analysis

Q. 8 The total exports and revenues from the exports of a country are given in the two pie charts below. The pie chart for exports shows the quantity of each item as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 5 lakh tonnes and the total revenues are 250 crore rupees. What is the ratio of the revenue generated through export of Item 1 per kilogram to the revenue generated through export of Item 4 per kilogram?



(A) 1 : 2 (B) 2 : 1

(C) 1 : 4 (D) 4 : 1

Q. 9 X is 1 km northeast of Y. Y is 1 km southeast of Z. W is 1 km west of Z. P is 1 km south of W. Q is 1 km east of P. What is the distance between X and Q in km?

(A) 1 (B) $\sqrt{2}$

(C) $\sqrt{3}$ (D) 2

Q. 10 10% of the population in a town is HIV^+. A new diagnostic kit for HIV detection is available; this kit correctly identifies HIV^+ individuals 95% of the time, and HIV^- individuals 89% of the time. A particular patient is tested using this kit and is found to be positive. The probability that the individual is actually positive is _______

END OF THE QUESTION PAPER

Civil Engineering

Q. 1 - Q. 25 Carry one mark each.

Q. 1 A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes : (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is

(A) 0 (B) 1/2

(C) 4/5 (D) 1/5

Sol. 1 Correct option is (B).

Every time a coin is tossed, its out come are independent of the previous outcomes. It may either result into a heads or a tails. So No. of favorable events (i.e. Tails) $= 1$

Total No. of outcomes (i.e. Head or Tail) $= 2$

So, Probability $= \frac{1}{2}$

Q. 2 The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is _____

Sol. 2 Correct answer is 88

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$$
$$C_1 \quad C_2 \quad C_3 \quad C_4$$

Determinant can be solved by calculating

$$0\begin{vmatrix} 0 & 3 & 0 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 3 & 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

or by doing column transformation as shown. Column transformation will not change the value of determinant So $C_3 \to C_3 - 2C_2$ & $C_4 \to C_4 - 3C_2$ given

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & -6 & -8 \\ 3 & 0 & 1 & 2 \end{vmatrix}$$

Solving determinant by taking row 1 (It has more no of 0)

$$\text{So } 0\begin{vmatrix} 0 & 3 & 0 \\ 3 & -6 & -8 \\ 0 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 & 0 \\ 2 & -6 & -8 \\ 3 & 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & -8 \\ 3 & 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & -6 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= -1(1 \times (-12 + 8) - 3(4 + 24) + 0(2 + 18))$$

$$= -1(-4 - 84) = 88$$

Q. 3 $z = \frac{2 - 3i}{-5 + i}$ can be expressed as

(A) $-0.5 - 0.5i$ (B) $-0.5 + 0.5i$

(C) $0.5 - 0.5i$ (D) $0.5 + 0.5i$

Sol. 3 Correct option is (B).

$$z = \frac{2-3i}{-5+i}$$

Simplifying this by multiplying & dividing term with conjugate of denominator

$$z = \frac{(2-3i)(-5-i)}{(-5+i)(-5-i)}$$

$$z = \frac{(2)\times(-5)-2i+15i+3(i)^2}{(-5)^2-(i)^2} \qquad ((a-b)(a+b) = a^2-b^2)$$

$$z = \frac{-10+13i-3}{25-(-1)} = \frac{-13+13i}{26} \qquad (\sqrt{-1} = i;\ i^2 = -1)$$

$$z = -\frac{1}{2}+\frac{1}{2}i = -0.5+0.5i$$

Q. 4 The integrating factor for the differential equation $\frac{dP}{dt}+k_2P = k_1L_oe^{-k_1t}$ is

(A) e^{-k_1t} (B) e^{-k_2t}

(C) e^{k_1t} (D) e^{k_2t}

Sol. 4 Correct option is (D).

Given equation

$$\frac{dP}{dt}+k_2P = k_1L_0e^{-k_1t}$$

The given equation is a linear differential equation of the form

$$\frac{dy}{dx}+Ay = B \qquad ...(1)$$

Where A & B are functions of (x), IF $= e^{\int A dx}$

Similarly comparing here with equation (1)

$$A = k_2 \ \& \ B = k_1L_0e^{-k_1t}$$

So Integrating factor $= e^{\int A dt} = e^{\int k_2 dt} = e^{k_2t}$

Q. 5 If $\{x\}$ is a continuous, real valued random variable defined over the interval $(-\infty,+\infty)$ and its occurrence is defined by the density function given as : $f(x) = \frac{1}{\sqrt{2\pi}*b}e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random

variable $\{x\}$. The value of the integral $\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}*b}e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}dx$ is

(A) 1 (B) 0.5

(C) π (D) $\pi/2$

Sol. 5 Correct option is (B).

$$\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}\times b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}dx = ?$$

Let $\frac{1}{\sqrt{2}}\frac{x-a}{b} = t$ for simplification of integration

So differentiating both sides

$$\frac{dx}{b\sqrt{2}} = dt \Rightarrow dx = \sqrt{2}\,dt\times(b) \qquad ...(1)$$

for change of limits when

$$\left.\begin{array}{ll} x = a & t = \frac{1}{\sqrt{2}}\times\frac{a-a}{b} = 0 \\ x = -\infty & t = \frac{1}{\sqrt{2}}\times\frac{-\infty-a}{b} = -\infty \end{array}\right\} \qquad ...(2)$$

& $$t^2 = \frac{1}{2}\left(\frac{x-a}{b}\right)^2 \qquad ...(3)$$

Substituting (1), (2) & (3) in

$$A = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi} \times b} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi} \times b} e^{-t^2} (dt \times \sqrt{2}\, b)$$

$$A = \frac{\sqrt{2} \times b}{\sqrt{2\pi} \times b} \int_{-\infty}^{0} e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-t^2} dt$$

Again $t^2 = z$ So $\qquad 2tdt = dz \Rightarrow dt = \frac{dz}{2t} = \frac{dz}{2}(z)^{-1/2}$

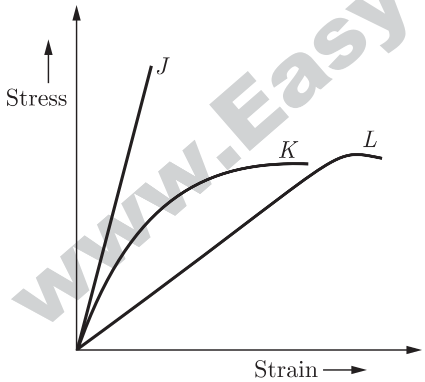
$t = 0;\ z = 0$

$t = -\infty;\ z = (-\infty)^2 = \infty$

So $$A = \frac{1}{\sqrt{\pi}} \int_{\infty}^{0} e^{-z} \frac{dz}{2}(z)^{-1/2} = \frac{1}{2\sqrt{\pi}} \int_{\infty}^{0} z^{-1/2} e^{-z} dz$$

$$= \frac{1}{2\sqrt{\pi}} \times \sqrt{\pi} = \frac{1}{2} = 0.5 \qquad \left(\int_{\infty}^{0} e^{-z} z^{-1/2} dz = \sqrt{\pi}\right)$$

Q. 6 Group I contains representative stress-strain curves as shown in the figure, while Group II gives the list of materials. Match the stress-strain curves with the corresponding materials.



	Group I		Group II
P.	Curve J	1.	Cement paste
Q.	Curve K	2.	Coarse aggregate
R.	Curve L	3.	Concrete

(A) P-1; Q-3; R-2 (B) P-2; Q-3; R-1

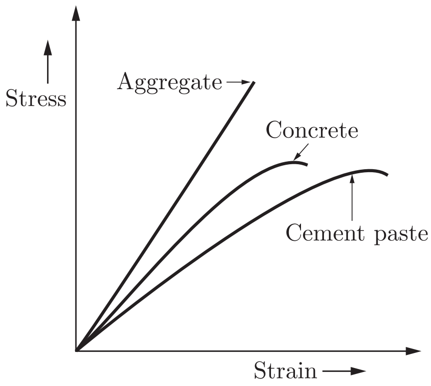
(C) P-3; Q-1; R-2 (D) P-3; Q-2; R-1

Sol. 6 Correct option is (B).

Aggregate behave as linear elastic material and it has a straight line curve.

Concrete gives less strain as compared to cement past for same amount of applied stress.

So,



Q. 7 The first moment of area about the axis of bending for a beam cross-section is

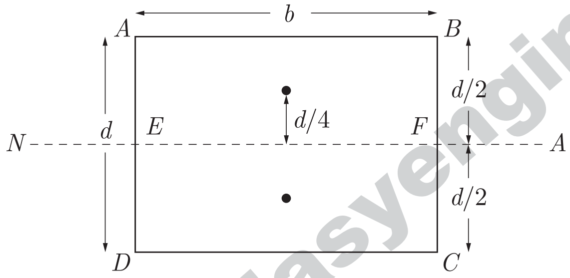
(A) moment of inertia (B) section modulus

(C) shape factor (D) polar moment of inertia

Sol. 7 Correct option is (B).

Section modulus Z is the first moment of area of beam cross section.

For example Plastic section modulus for rectangular beam cross section.



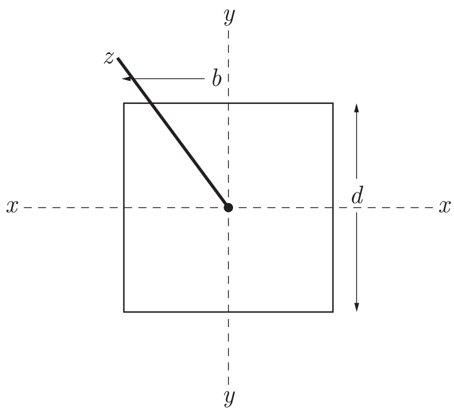
$Z_p =$ Area (AEFB) $\times$ distance of centroid of area from neutral axis + Area (DEFC) $\times$ distance of centroid of area from neutral axis

$$= \left(\frac{bd}{2}\right) \times \left(\frac{d}{4}\right) + \left(\frac{bd}{2}\right) \times \frac{d}{4} = \frac{bd^2}{4}$$

Q. 8 Polar moment of inertia (I_p), in cm^4, of a rectangular section having width, $b = 2$ cm and depth, $d = 6$ cm is ______

Sol. 8 Correct answer is 40

If x, y, z are three mutually perpendicular direction.



Polar moment of inertia is the moment of inertia about axis perpendicular to cross section i.e. I_z

$$I_p = I_z = I_x + I_y$$

for rectangular cross section

$$I_x = \frac{bd^3}{12}, \quad I_y = \frac{b^3 d}{12}$$

here $b = 2\,\text{m},\ d = 6\,\text{cm}$

So $$I_p = I_z = \frac{2\times 6^3}{12} + \frac{2^3\times 6}{12} = \frac{6^3}{6} + \frac{2^3}{2} = 6^2 + 2^2 = 36 + 4$$

$$= 40\,\text{cm}^4$$

Q. 9 The target mean strength f_{cm} for concrete mix design obtained from the characteristic strength f_{ck} and standard deviation σ, as defined in IS : 456-2000, is

(A) $f_{ck} + 1.35\sigma$

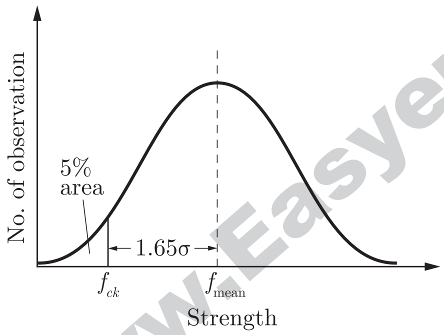
(B) $f_{ck} + 1.45\sigma$

(C) $f_{ck} + 1.55\sigma$

(D) $f_{ck} + 1.65\sigma$

Sol. 9 Correct option is (D).

Concrete cubes when tested for strength forms a normal distribution curve as shown



Characteristic strength is given by, concrete strength below which not more than 5% of test results fall

So from graph

$$f_{\text{mean}} = f_{ck} + 1.65\sigma$$

where σ is standard deviation from mean.

Q. 10 The flexural tensile strength of M25 grade of concrete, in N/mm^2, as per IS : 456-2000 is _____.

Sol. 10 Correct answer is 3.5

According to IS 456 2000 flexural tensile strength is given as $0.7\sqrt{f_{cx}}$ (clause 6.2.2) Here $f_{ck} = 25$

So Tensile Strength $= 0.7\sqrt{25} = 3.5$

Q. 11 The modulus of elasticity, $E = 5000\sqrt{f_{ck}}$ where f_{ck} is the characteristic compressive strength of concrete, specified in IS : 456-2000 is based on

(A) tangent modulus

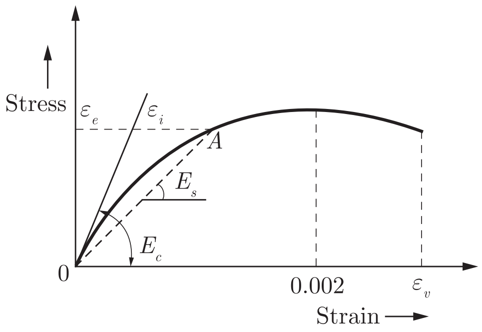
(B) initial tangent modulus

(C) secant modulus

(D) chord modulus

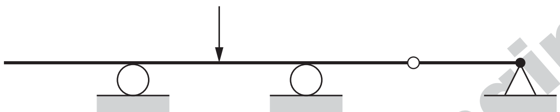
Sol. 11 Correct option is (B).

Stress strain curve for concrete is as shown below

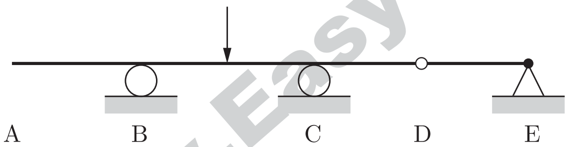


ε_e is elastic strain at A
ε_i is inelastic strain at A
So slope of curve in the beginning of the stress strain curve (i.e. tangent to it) is known as initial tangent modulus and is given by $E_c = 5000\sqrt{f_{cx}}$ (Acc to IS 456 : 2000)
Slope of OA is known as secant modulus E_s.

Q. 12 The static indeterminacy of the two-span continuous beam with an internal hinge, shown below, is



Sol. 12 Correct answer is 0



Static indeterminacy "D_S" is the no. of unknown reactions in excess of available equilibrium equations.

So D_S = No. of unknown reactions – Available equilibrium equations

$= R - E$

R = No. of unknown reaction = 1 (at B in vertical dir. due to roller)

+1 (at C in vertical direction due to roller) +2 (1 in horizontal & 1 vertical direction at E due to hinged support)

$R = 4$

E = No. of equilibrium equations

$= 3(\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0) + 1$ ($\Sigma M_D = 0$ due to pin or hing at D)

$E = 4$

So $D_S = 4 - 4 = 0$

or by Formula

$$D_S = 3m + R - R' - 3j$$

m = total number of member = 4

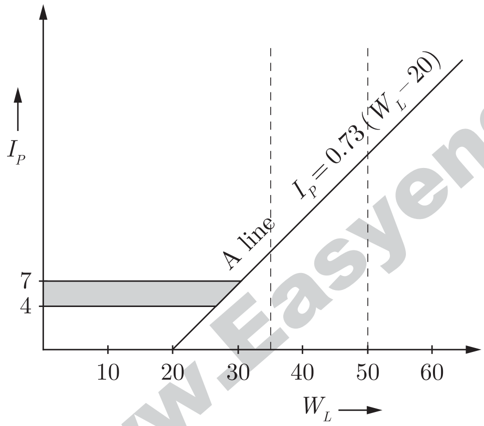
$$R = \text{No. of reaction} = 4$$
$$R' = \text{additional equation available} = 1 \ (M_D = 0)$$
$$j = \text{Total joints} = 5 \ (A, B, C, D, E)$$
$$D_S = 3 \times 4 + 4 - 1 - 3 \times 5$$
$$= 0$$

Q. 13 As per Indian Standard Soil Classification System (IS : 1498-1970), an expression for A-line is

(A) $I_p = 0.73(w_L - 20)$
(B) $I_p = 0.70(w_L - 20)$
(C) $I_p = 0.73(w_L - 10)$
(D) $I_p = 0.70(w_L - 10)$

Sol. 13 Correct option is (A).

As per (IS : 1498-1970) the line which separates silt fraction with clay fraction is known as A-line and has the equation of $I_P = 0.73(W_L - 20)$ in a plot b/w plasticity index (I_P) and Liquid limit (W_L)



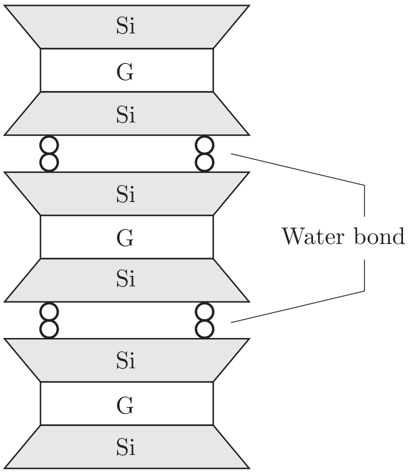
The portion below A-line is silt fraction & above is clay fraction.

Q. 14 The clay mineral primarily governing the swelling behavior of Black Cotton soil is

(A) Halloysite
(B) Illite
(C) Kaolinite
(D) Montmorillonite

Sol. 14 Correct option is (D).

Clay mineral montmorillonite is present in Black cotton soil which increases in volume when absorbs water. It is composed of two silica and one gibbsite (Alumina) sheet as shown



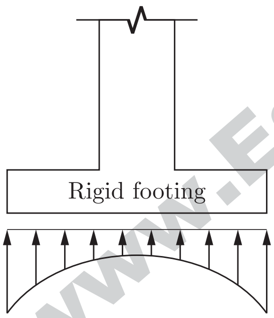
The inter layer bonding is due to weak vanderwall forces and surface area is also large which is responsible for its swelling characteristic.

Q. 15 The contact pressure for a rigid footing resting on clay at the centre and the edges are respectively

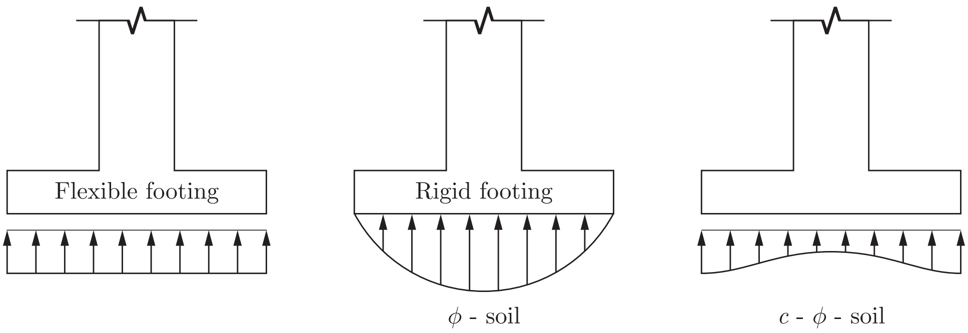
(A) maximum and zero
(B) maximum and minimum
(C) zero and maximum
(D) minimum and maximum

Sol. 15 Correct option is (D).

The contact pressure distribution for a rigid footing on a clay is as shown



Pressure is maximum at edges and minimum but not zero at centre.
Theoretically, it is uniform for flexible footing irrespective of type of soil.



Q. 16 A certain soil has the following properties : $G_s = 2.71$, $n = 40\%$ and $w = 20\%$. The degree of saturation of the soil (rounded off to the nearest percent) is ____

Sol. 16 Correct answer is 81.3

Given $G_s = 2.71$ (Specific Gravity)

$n = 40\%$ or $n = 0.40$ (ratio form) – porosity

$w = 20\%$ or $w = 0.20$ (ratio form) – water content

relation ship b/w void ratio e, G_s, w & S_r (Degree of saturation) is

$$e = \frac{wG_s}{S_r} \qquad ...(1)$$

& $$e = \frac{n}{1-n} = \frac{0.40}{1-0.40} = \frac{0.40}{0.60} = \frac{2}{3}$$

Now from (1) $$S_r = \frac{wG_s}{e} = \frac{0.20 \times 2.71}{(2/3)} = 0.813$$

Expressing in percentage $S_r = 81.3\%$

Q. 17 A plane flow has velocity components $u = \frac{x}{T_1}$, $v = -\frac{y}{T_2}$ and $w = 0$ along x, y and z directions respectively, where $T_1 (\neq 0)$ and $T_2 (\neq 0)$ are constants having the dimension of time. The given flow is incompressible if

(A) $T_1 = -T_2$

(B) $T_1 = -\frac{T_2}{2}$

(C) $T_1 = \frac{T_2}{2}$

(D) $T_1 = T_2$

Sol. 17 Correct option is (D).

Continuity equation is given as

$$\frac{\partial p}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

where ρ is density of fluid and u, v, w are velocity in x, y & z direction respectively.

But for plane incompressible flow ρ remain unchanged So

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad ...(i)$$

So putting $u = \frac{x}{T_1}$ & $v = \frac{y}{T_2}$ in (i)

$$\frac{1}{T_1} - \frac{1}{T_2} = 0 \Rightarrow T_1 = T_2$$

Q. 18 Group I lists a few devices while Group II provides information about their uses. Match the devices with their corresponding use.

	Group I		Group II
P.	Anemometer	1.	Capillary potential of soil water
Q.	Hygrometer	2.	Fluid velocity at a specific point in the flow stream
R.	Pitot Tube	3.	Water vapour content of air
S.	Tensiometer	4.	Wind speed

(A) P-1; Q-2; R-3; S-4

(B) P-2; Q-1; R-4; S-3

(C) P-4; Q-2; R-1; S-3

(D) P-4; Q-3; R-2; S-1

Sol. 18 Correct option is (D).

Anemometer measures wind speed.

Hygrometer is used to measure humidity or water vapour content in air. Pitot tube measures fluid velocity at a specific point in flow stream using bernoulli's equation.

Tensiometer measure capillary potential or soil suction in soil.

Q. 19 An isolated 3-h rainfall event on a small catchment produces a hydrograph peak and point of inflection on the falling limb of the hydrograph at 7 hours and 8.5 hours respectively, after the start of the rainfall. Assuming, no losses and no base flow contribution, the time of concentration (in hours) for this catchment is approximately

(A) 8.5 (B) 7.0

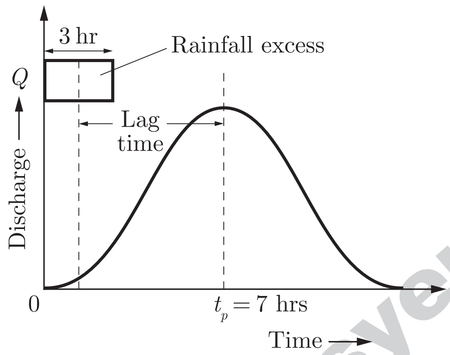
(C) 6.5 (D) 5.5

Sol. 19 Correct option is (D).

Time of concentration is the time taken by water to flow from most remote point, to a watershet outlet. For a small catchment, time of concentration is approximately equal to Lag time.

Lag time, which is defined as the time from C.G of Rainfall excess to the peak of Hydrograph is : T_L as shown in the Hydrograph.

Given Rainfall excess $= 3\,hr$ & Peak time $= 7$ hrs



From above lag time $T_L = 7 - \frac{3}{2} = 5.5$ hrs

& Here $T_C = T_L = 5.5$ hrs

Q. 20 The Muskingum model of routing a flood through a steam reach is expressed as $O_2 = K_0 I_2 + K_1 I_1 + K_2 O_1$, where K_0, K_1 and K_2 are the routing coefficients for the concerned reach, I_1 and I_2 are the inflows to the reach, and O_1 and O_2 are the outflows from the reach corresponding to time steps 1 and 2 respectively. The sum of K_0, K_1 and K_2 of the model is

(A) -1 (B) -0.5

(C) 0.5 (D) 1

Sol. 20 Correct option is (D).

Muskingum model is used for channel Routing. According to muskingum model

$$Q_2 = K_0 I_2 + K_1 I_1 + K_2 Q_1$$

K_0, K_1 & K_2 being routing coefficients & $K_0 + K_1 + K_2 = 1$

If s is storage time coefficient & x is weighing factor, Δt is time interval.

$$K_0 = \frac{0.5\,\Delta t - sx}{s(1-x) + 0.5\,\Delta t} \quad K_1 = \frac{0.5\,\Delta t + sx}{s(1-x) + 0.5\,\Delta t} \quad K_2 = \frac{s(1-x) - 0.5\,\Delta t}{s(1-x) + 0.5\,\Delta t}$$

Q. 21 The dominating microorganisms in an activated sludge process reactor are

(A) aerobic heterotrophs (B) anaerobic heterotrophs

(C) autotrophs (D) phototrophs

Sol. 21 Correct option is (D).

Activated sludge process is suspended growth technique for treatment of waste water.

In activated sludge process wastewater containing organic matter is aerated in an aeration basin in which microorganism metabolise the organic particles. These microorganism act in presence of oxygen hence they are aerobic hetrotrophs.

Q. 22 The two air pollution control devices that are usually used to remove very fine particles from the flue gas are

(A) Cyclone and Venturi Scrubber

(B) Cyclone and Packed Scrubber

(C) Electrostatic Precipitator and Fabric Filter

(D) Settling Chamber and Tray Scrubber

Sol. 22 Correct option is (C).

Electrostatic precipitators and fabric filters/baghouse are used to remove very fine particles like fly cash from fuel gases of the industries and municipal incinerators etc.

Cyclone, Settling chamber & scrubber etc are used to remove relatively larger particles.

Q. 23 The average spacing between vehicles in a traffic stream is 50 m, then the density (in veh/km) of the stream is __________

Sol. 23 Correct answer is 20

Vehicle density is the total number of vehicle in unit length of road.

Given spacing $= 50\,\text{m}$

So 1 vehicle is there at every $50\,\text{m}$ spacing.

So, η vehicles are there in 1 km or 1000 m length

So $$\frac{\eta}{1} = \frac{1000}{50} \Rightarrow \eta = \frac{1000}{50} = 20\,\text{Veh}$$

So density $= 20\,\text{vehicle/km}$

or Alternatively

$$K = \text{Density} = \frac{1000}{s}\,\text{veh/km}$$

where s is spacing in m

$$\text{Density} = \frac{1000}{50} = 20\,\text{veh/km}$$

Q. 24 A road is being designed for a speed of $110\,\text{km/hr}$ on a horizontal curve with a super elevation of 8%. If the coefficient of side friction is 0.10, the minimum radius of the curve (in m) required for safe vehicular movement is

(A) 115.0 (B) 152.3

(C) 264.3 (D) 528.5

Sol. 24 Correct option is (D).

Ruling minimum radius is given by

$$R = \frac{V^2}{127(e+f)} \quad \left\{\text{from } (f+e) = \frac{v^2}{gr}\right\}$$ where v is in m/s

& r is in m

Velocity in km/hr $V = 110\,\text{km/hr}$

Ruling minimum radius, $R = ?$

$$\text{Super elevation in ratio, } e = 0.08$$

$$\text{Coefficient of lateral friction, } f = 0.10$$

So $$R = \frac{(110)^2}{127(0.08+0.10)} = \frac{(110)^2}{127 \times 0.18} = 529.3\,\text{m}$$

Nearest option is 528.5.

Q. 25 The survey carried out to delineate natural features, such as hills, rivers, forests and man-made features, such as towns, villages, buildings, roads, transmission lines and canals is classified as

(A) engineering survey
(B) geological survey
(C) land survey
(D) topographic survey

Sol. 25 Correct option is (D).

A topographical survey is defined as a survey which involves a map showing general topography along with natural features like hills, rivers forest, town village etc.

Land survey in addition to topographical survey also includes fixing property lines.

Engineering survey is carried out for engg. works such as roads, railway reservoir etc.

Q. 26 - Q. 55 Carry two marks each.

Q. 26 The expression $\lim\limits_{\alpha \to 0} \frac{x^a - 1}{\alpha}$ is equal to

(A) $\log x$
(B) 0
(C) $x \log x$
(D) ∞

Sol. 26 Correct option is (A).

$$\lim_{a \to 0} \frac{x^a - 1}{a}$$

Putting limit $a \to 0$ we get

$$\frac{x^0 - 1}{0} = \frac{0}{0} \text{ form}$$

So L-hospital rule can be applied

$$\lim_{a \to 0} \frac{\frac{\partial}{\partial a}(x^a - 1)}{\frac{\partial}{\partial a}(a)} = \lim_{a \to 0} \frac{x^a \ln x}{1} = \frac{x^0 \log x}{1}$$

$$= \log x$$

(here x is being treated like constant)

Q. 27 An observer counts $240\,\text{veh/h}$ at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30 second time interval is _ _ _ _ _ _ _

Sol. 27 Correct answer is 0.270

The probability of observing x events in a given interval for possion's distribution is

$$P(X = x) = \frac{e^{-\lambda}(\lambda)^x}{x\,!}$$

Where, λ is mean number of events per interval

Given mean volume $= 240 \text{ vehicle/hr} = \frac{240}{60 \times 60} = \frac{1}{15} \text{ veh/sec}$

So mean number of vehicle in 30 sec interval $(\lambda) = \frac{30}{15} = 2$

$x = 1$ (We have to find probability of one vehicle per 30 sec)

So Probability $P(X = 1) = \frac{e^{-2}(2)^1}{1!} = 0.270$

Q. 28 The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is ________

Sol. 28 Correct answer is 2

Rank of a matrix A is defined as the number of linearly independent column vectors of A or number of linearly independent row vectors of A.

No. of linearly independent Row = No. of linearly independent column

Here Given $A = \begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -18 \end{bmatrix} \begin{matrix} -R_1 \\ -R_2 \\ -R_3 \end{matrix}$

Applying row transformations

$$R_3 \to R_3 + R_2 \begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 12 & 0 & 8 & 0 \end{bmatrix} \begin{matrix} -R_1 \\ -R_2 \\ -R_3 \end{matrix}$$

$$R_3 \to R_3 - 2R_1 \begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Two rows are linearly independent So Rank = 2.

Q. 29 Water is flowing at a steady rate through a homogeneous and saturated horizontal soil strip of 10 m length. The strip is being subjected to a constant water head (H) of 5 m at the beginning and 1 m at the end. If the governing equation of flow in the soil strip is $\frac{d^2H}{dx^2} = 0$ (where x is the distance along the soil strip), the value of $H(\text{in } m)$ at the middle of the strip is _______

Sol. 29 Correct answer is 3

Given governing equation of flow

$$\frac{d^2H}{dx^2} = 0 \qquad \text{...(1)}$$

x is distance along strip and H is head at x

Given boundary condition at starting

$$x = 0 \qquad H = 5$$

at end $\qquad x = 10 \qquad H = 1$

integrating eq (1)

$$\int \frac{d^2H}{dx^2}dx = \int 0\, dx$$

$$\frac{dH}{dx} = C_1 \qquad (C_1 \text{ is constant})$$

Again integrating $\quad \int \frac{dH}{dx}dx = \int C_1\, dx$

$$H + C_2 = C_1 x \qquad \text{...(2)}$$

C_2 is any constant

Putting boundary condition in (2)

$$5 + C_2 = C_1 \times 0 \Rightarrow C_2 = -5$$

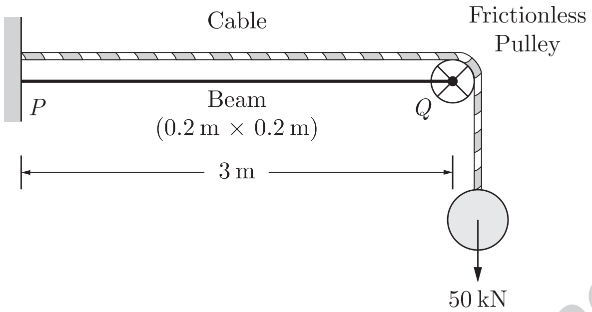
$$1 + (-5) = C_1 \times 10 \Rightarrow C_1 = -0.4$$

So Now eq (2) is $H - 5 = -0.4x$

Now at $x = 5$; $H - 5 = -0.4 \times 5$

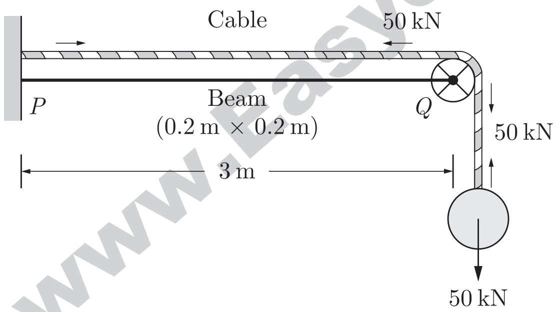
So $H = 5 - 2 = 3\,\text{m}$

Q. 30 The values of axial stress (σ) in kN/m^2, bending moment (M) in kNm, and shear force (V) in kN acting at point P for the arrangement shown in the figure are respectively.



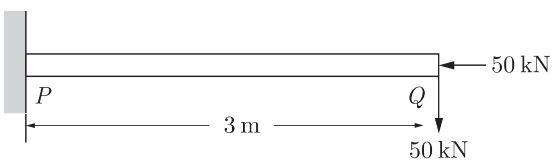
(A) 1000, 75 and 25
(B) 1250, 150 and 50
(C) 1500, 225 and 75
(D) 1750, 300 and 100

Sol. 30 Correct option is (B).



The cable has uniform tension 50 kN over its length due to hanging mass of 50 kN mass

Resultant forces on beam

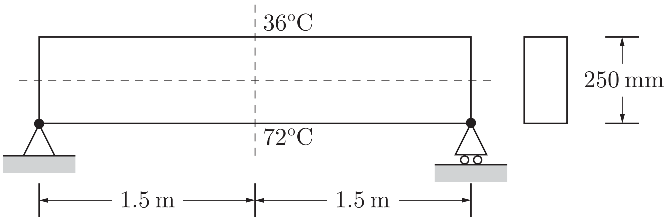


So stress in cross section $\sigma = \dfrac{\text{force}}{\text{Area}} = \dfrac{50\,\text{kN}}{0.2 \times 0.2} = 1250\,\text{km/m}^2$

Shear force at P $F_s = 50\,\text{kN}$

Moment at P, $M = 50 \times PQ = 50 \times 3 = 150\,\text{kNm}$

Q. 31 The beam of an overall depth 250 mm (shown below) is used in a building subjected to two different thermal environments. The temperatures at the top and bottom surfaces of the beam are 36°C and 72°C respectively. Considering coefficient of thermal expansion (α) as 1.50×10^{-5} per $^\circ$C, the vertical deflection of the beam (in mm) at its mid-span due to temperature gradient is _ _ _ _ _.



Sol. 31 Correct answer is 2.43

Given

$$\text{top surface temperature} = 36^\circ\text{C}$$

$$\text{Bottom surface temp} = 72^\circ\text{C}$$

'α' is coefficient of thermal expansion

$$= 1.50 \times 10^{-5} \text{ per } ^\circ\text{C} \ \& \ \text{Length of beam } L = 3\text{ m}$$

So,

Average change in temperature,

$$T = \frac{72 - 36}{2} = \frac{36}{2} = 18^\circ\text{C}$$

Now, average change in length of beam

$$\Delta L = L\alpha T$$

& Strain, $\varepsilon_0 = \frac{L\alpha T}{L} = \alpha t$ ...(i)

So, By pure bending equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

So curvature $\frac{1}{R} = \left(\frac{f}{E}\right)\frac{1}{y} = \frac{\varepsilon_0}{y}$ ...(ii)

$$\left(\frac{f}{E} = \frac{\text{Stress}}{\text{Young's modulus}} = \text{Strain} = \varepsilon_0\right)$$

By properly of circle, deflection δ is given by

$$\delta = \frac{L^2}{8R}$$

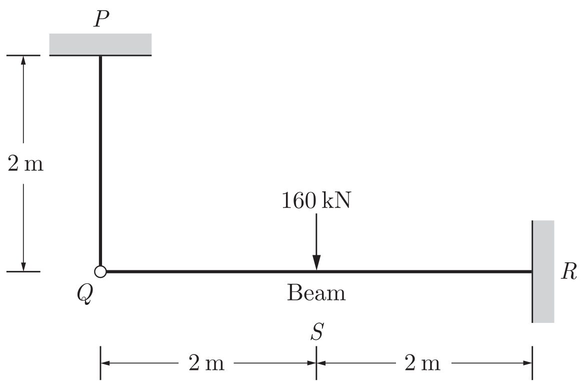
Using (i) & (ii) we get

$$\delta = \frac{(3)^2(\varepsilon_0)}{8 \times (y)} = \frac{(3)^2(1.50 \times 10^{-5} \times 18)}{8 \times (0.250/2)}$$

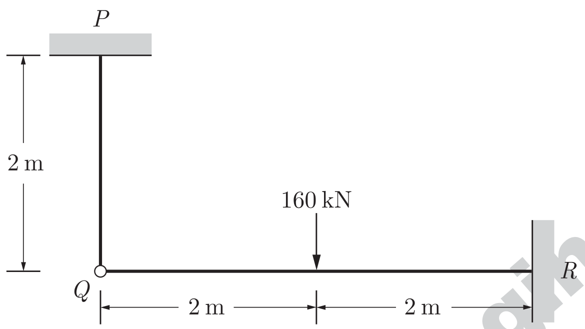
$$= 2.43 \times 10^{-3}\text{ m}$$

$$\text{in mm deflection} = 2.43\text{ mm}$$

Q. 32 The axial load (in kN) in the member PQ for the arrangement/assembly shown in the figure given below is_ _ _ _ _ _



Sol. 32 Correct answer is 50



Applying slope deflection method and assuming numbers to be inextensible

for QR fix end moment $M^F_{QR} = -\frac{wL}{8} = -\frac{160\times 4}{8} = -80$ kNm

fix end moment $M^F_{RQ} = +\frac{wL}{8} = \frac{160\times 4}{8} = +80$ kNm

Slope deflection equation

$$M_{QR} = M^F_{QR} + \frac{2EI}{L}(2\theta_Q + \theta_R)$$

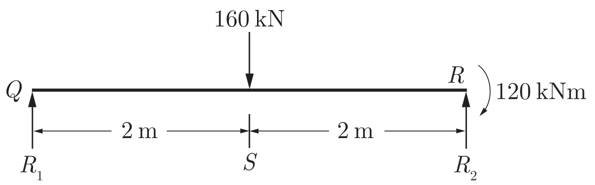
$$= -80 + \frac{2EI}{4}(2\theta_Q) \qquad (R \text{ is fixed So } \theta_R = 0)$$

& $M_{QR} = 0$ $\qquad \theta_Q = \frac{80}{EI} \qquad (Q \text{ is hinged}) \quad ...(i)$

Similarly $\qquad M_{RQ} = M^F_{RQ} + \frac{2EI}{L}(2\theta_R + \theta_Q)$

$$= 80 + \frac{2EI}{4}\left(\frac{80}{EI}\right) = 120 \text{ kNm} \qquad (\text{from (1)})$$

FBD for Beam QR



Let R_1 force act at Q

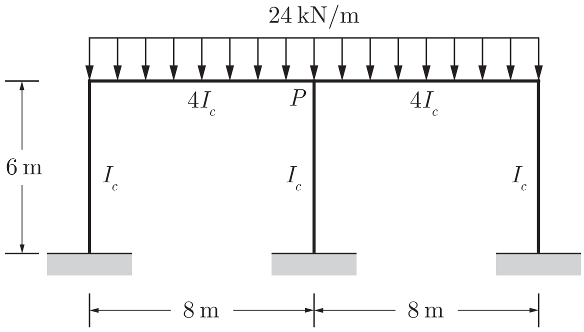
So $\qquad \Sigma M_R = 0$

$$R_1 \times 4 - 160 \times 2 + 120 = 0$$

$$R_1 = 50 \text{ kN}$$

Axial force is equal to $\qquad R_1 = 50$ kN in PQ by equilibrium.

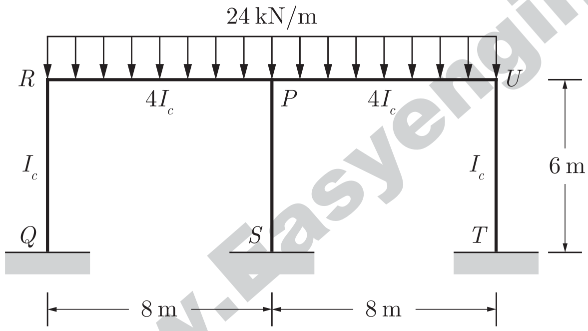
Q. 33 Considering the symmetry of a rigid frame as shown below, the magnitude of the bending moment (in kNm) at P (preferably using the moment distribution method) is



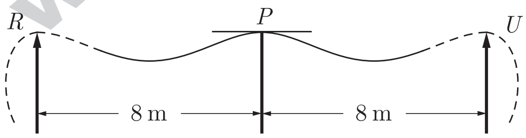
(A) 170 (B) 172

(C) 176 (D) 178

Sol. 33 Correct option is (C).



Span RU is uniformly loaded and span RP & PU one equal in length so joint P will act as fixed with $\theta_P = 0$ as shown



Span RP, Fix end moment at R

$$M_{RP}^F = -\frac{wl^2}{12} = -\frac{24 \times 8^2}{12} = -128$$

Fix end moment at P, $M_{PR}^F = \frac{wl^2}{12} = 128$

So slope deflection eqn for RP

$$M_{RP} = M_{RP}^F + \frac{2E(4I_C)}{L}(2\theta_R + \theta_P)$$

$$= -128 + \frac{8EI_C}{8} \times (2\theta_R)$$

$$= -128 + 2EI_C\theta_R \quad \text{...(i)}$$

$$M_{PR} = M_{PR}^F + \frac{2E(4I_C)}{L}(2\theta_P + \theta_R)$$

$$= 128 + \frac{8EI_C}{8}(\theta_R) \quad \text{...(ii)}$$

for RQ $M^F_{RQ} = 0$ $\quad M_{RQ} = \frac{2EI_C}{6} \times (2\theta_R + \theta_Q) = \frac{2EI_C}{3}\theta_R$ ...(iii)

Equilibrium eq Now $\Sigma M_R = 0$

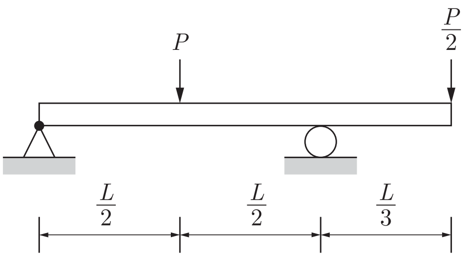
$M_{RP} + M_{RQ} = 0$ $\quad -128 + \left(2EI_C + \frac{2EI_C}{3}\right)\theta_R = 0$

$$\theta_R = \frac{48}{EI_C} \quad \text{...(iv)}$$

putting (iv) in (ii)

$$M_{PR} = 128 + \frac{8EI_C}{8}\left(\frac{48}{EI_C}\right) = 176\,\text{kNm is moment at } P$$

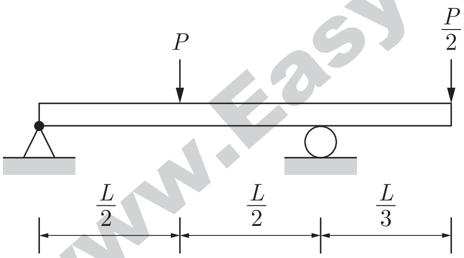
Q. 34 A prismatic beam (as shown below) has plastic moment capacity of M_p, then the collapse load P o f the beam is



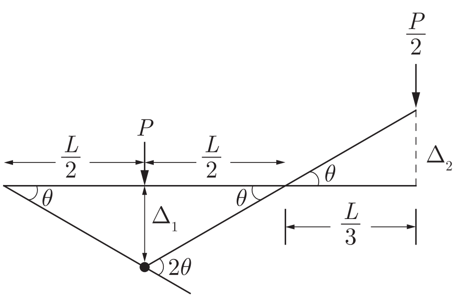
(A) $\frac{2M_p}{L}$ (B) $\frac{4M_p}{L}$

(C) $\frac{6M_p}{L}$ (D) $\frac{8M_p}{L}$

Sol. 34 Correct option is (C).



The collapse of beam will take place by formation of plastic hing below load P. By kinematic method of plastic analysis, deflected beam shape is



So $\quad \Delta_1 = \theta \times \frac{L}{2} = \frac{\theta L}{2}$ & $\Delta_2 = \theta \times \frac{L}{3} = \frac{\theta L}{3}$

So internal work done = External work done

$$M_P(2\theta) = P \times (\Delta_1) - \frac{P}{2}(\Delta_2)$$

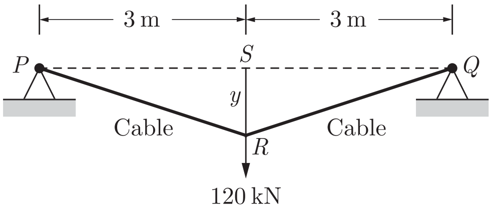
($-$ve sign for Δ_2 because it is opposite to direction of $P/2$)

So $$M_P \times (2\theta) = P \times \frac{\theta L}{2} - \frac{P}{2} \times \frac{\theta L}{3}$$

$$2M_P = PL \times \left(\frac{2}{6}\right)$$

So $$P = \frac{6M_P}{L}$$

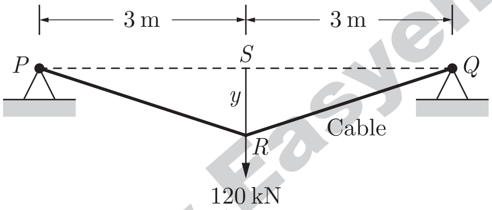
Q. 35 The tension (in kN) in a 10 m long cable, shown in the figure, neglecting its self-weight is



(A) 120 (B) 75

(C) 60 (D) 45

Sol. 35 Correct option is (B).



$$\text{Total length of cable} = 10\text{ m}$$

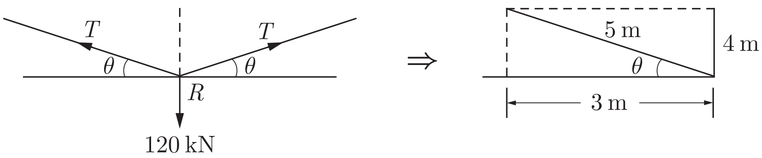
So $$PR = QR = 5\text{ m}$$

Both the portion of cable will have uniform tension say 'T'

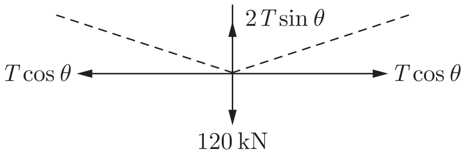
Now $$y^2 = PR^2 - RS^2$$

$$y = \sqrt{5^2 - 3^2} = 4\text{ m}$$

FBD (free body diagram) for cable at R



breaking into components

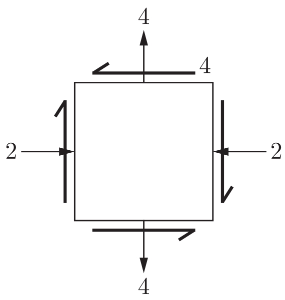


So $\Sigma y = 0 \Rightarrow$ $$2T\sin\theta = 120$$

$$T = \frac{120}{2\sin\theta} = \frac{120}{2 \times (4/5)}$$

So $$T = 75\text{ kN}$$

Q. 36 For the state of stresses (in MPa) shown in the figure below, the maximum shear stress (in MPa) is

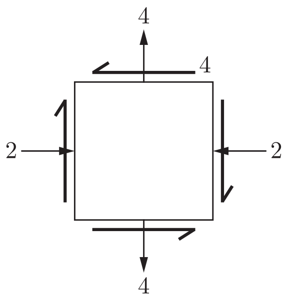


Sol. 36 Correct answer is 5

If σ_1 & σ_2 are principle stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

For the static of stress



$\sigma_x = -2, \quad \sigma_y = 4, \quad \tau_{xy} = 4$

$$\sigma_1, \sigma_2 = \frac{-2+4}{2} \pm \sqrt{\left(\frac{-2-4}{2}\right)^2 + (4)^2}$$

$$= 1 \pm \sqrt{9+16} = 1 \pm 5$$

So $\sigma_1 = 6\,\text{MPa}$ & $\sigma_2 = -4\,\text{MPa}$

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{6-(-4)}{2} = 5\,\text{MPa}$

Q. 37 An infinitely long slope is made up of a $c - \varphi$ soil having the properties : cohesion $(c) = 20\,\text{kPa}$, and dry unit weight $(\gamma_d) = 16\,\text{kN/m}^3$. The angle of inclination and critical height of the slope are 40° and $5\,\text{m}$, respectively. To maintain the limiting equilibrium, the angle of internal friction of the soil (in degrees) is _____

Sol. 37 Correct answer is 23.73

Given $c = 20\,\text{kPa} \quad \gamma_d = 16\,\text{kN/m}^3$

Critical Height $H_C = 5\,\text{m}$ & angle of slope, $i = 40^\circ$

Angle of internal friction $\phi = ??$

So for $c - \phi$ soil factor of safety

$$F = \frac{c + \sigma \tan\phi \cos^2 i}{\sigma \cos i \sin i} \frac{(\text{Resisting Moment})}{(\text{Driving moment})}$$

$\sigma = \gamma z$
$$F = \frac{c + \gamma_d z \tan\phi \cos^2 i}{\sigma \cos i \sin i} \qquad \text{...(i)}$$

for equilibrium $F = 1$ & $z = H_C$ eq (1) becomes

So $$H_C = \frac{c}{\gamma(\tan i - \tan\phi)\cos^2 i}$$ putting values

$$5 = \frac{20}{16(\tan 40 - \tan\phi)\cos^2 40}$$

$$\tan 40 - \tan\phi = 0.42$$

$$\tan\phi = 0.83 - 0.42 = 0.41$$

$$\phi = \tan^{-1}(0.41) = 22.73$$

Q. 38 Group I enlists in-situ field tests carried out for soil exploration, while Group II provides a list of parameters for sub-soil strength characterization. Match the type of tests with the characterization parameters.

	Group I		Group II
P.	Pressuremeter Test (PMT)	1.	Menard's modulus (E_m)
Q.	Static Cone Penetration Test (SCPT)	2.	Number of blows (N)
R.	Standard Penetration Test (SPT)	3.	Skin resistance (f_c)
S.	Vane Shear Test (VST)	4.	Undrained cohesion (c_u)

(A) P-1; Q-3; R-2; S-4

(B) P-1; Q-2; R-3; S-4

(C) P-2; Q-3; R-4; S-1

(D) P-4; Q-1; R-2; S-3

Sol. 38 Correct option is (A).

- Pressuremeter test gives the Menard's modulus (E_m) which is used for calculation of settlement of foundation.
- Standard cone Penetration test can also be used to determine skin friction (f_c) by tension load cell attached to the apparatus.
- In standard Penetration test number of blow required to advance the sampler tube into the soil for 30 cm is measured.
- Vane shear test measures the in site shear strength of soil, which is used to calculate undrained cohesion. (c_v)

Q. 39 A single vertical friction pile of diameter 500 mm and length 20 m is subjected to a vertical compressive load. The pile is embedded in a homogeneous sandy stratum where: angle of internal friction $(\varphi) = 30°$, dry unit weight $(\gamma_d) = 20\ \text{kN/m}^3$ and angle of wall friction $(\delta) = 2\varphi/3$. Considering the coefficient of lateral earth pressure $(K) = 2.7$ and the bearing capacity factor $(N_q) = 25$, the ultimate bearing capacity of the pile (in kN) is _____

Sol. 39 Correct answer is 6171.356

Given

diameter of pile $d = 500\ \text{mm} = 0.5\ \text{m}$

Length $L = 20\ \text{m}$; internal friction $\phi = 30°$;

dry unit weight $\gamma_d = 20$

Angle of wall friction $\delta = \frac{2\phi}{3} = \frac{2\times 30}{3} = 20°$

Lateral earth friction $k = 2.7$; Bearing capacity factor $N_q = 25$

Ultimate load bearing capacity of pile

$$= \text{Point Bearing} + \text{Skin friction}$$

$$= A_p S_p + A_s S_s$$

A_p = cross sectional area of pile $= \frac{\pi}{4}\times(d)^2 = \frac{\pi}{4}\times(0.5)^2 = 0.196\ \text{m}^2$

$A_s =$ Lateral area of pile $= \pi dL = \pi \times 0.5 \times 20 = 31.4\ \text{m}^2$

$S_p =$ Point Bearance per unit area

$$= \overline{\sigma} N_q = \gamma_d L N_q = 20 \times 20 \times 25 = 10000\ \text{kN/m}^2$$

$S_s =$ Skin friction per unit area $= \frac{1}{2} k\overline{\sigma} \tan\delta$

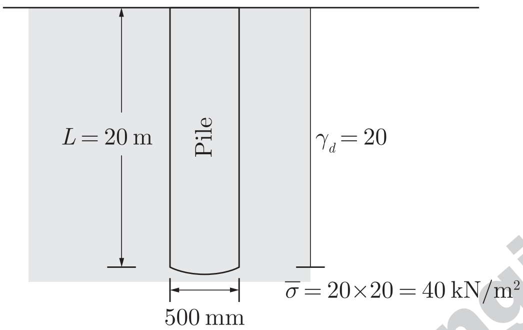
$$= \frac{1}{2} \times (2.7)(20 \times 20) \tan 20$$

$$= 196.54\ \text{kN/m}^2$$

So Point Bearance $= A_p S_p = 0.1962 \times 10000 = 1962\ \text{kN}$

Skin friction $= A_s S_s = 31.4 \times 196.54 = 6171.356\ \text{kN}$

It is sandy soil point bearance can be neglected so ultimate load bearing capacity $q_0 = A_s S_s = 6171.356\ \text{kN}$



Q. 40 A circular rate foundation of 20 m diameter and 1.6 m thick is provided for a tank that applies a bearing pressure of 110 kPa on sandy soil with Young's modulus, $E_s' = 30$ MPa and Poisson's ratio, $v_s = 0.3$. The raft is made of concrete ($E_c = 30$ GPa and $v_c = 0.15$). Considering the raft as rigid, the elastic settlement (in mm) is

(A) 50.96 (B) 53.36

(C) 63.72 (D) 66.71

Sol. 40 Correct option is (B).

Elastic settlement or immediate settlement for a foundation is given as

$$S_i = \frac{qB(1-\nu_s^2)}{E_s} I$$

where

'q' is bearing pressure of soil $= 110\ \text{kP}_a = 110 \times 10^{-3}\ \text{N/mm}^2$

'B' is width of footing $= 20\ \text{m} = 20 \times 10^3\ \text{mm}$

'ν_s' is poission's raho for soil $= 0.3$

'E_s' is elastic modulus for soil $= 30$ MPa or $30\ \text{N/mm}^2$

I_s is influence factor for rigid circular; $I_s = 0.8$

So Settlement $S_i(\text{mm}) = \frac{110 \times 10^{-3} \times 20 \times 10^3 (1-0.3)^2}{30} \times 0.8$

$$= 53.38\ \text{mm}$$

Q. 41 A horizontal nozzle of 30 mm diameter discharges a steady jet of water into the atmosphere at a rate of 15 litres per second. The diameter of inlet to the nozzle is 100 mm. The jet impinges normal to a flat stationary plate held close to the nozzle end. Neglecting air friction and considering the density of water as

1000 kg/m^3, the force exerted by the jet (in N) on the plate is ________

Sol. 41 Correct answer is 318.30

By impulse momentum theorem, force exerted by jet on the plate is,

$$F = \rho_1 Q_1 V_1 - \rho_2 Q_2 V_2$$

ρ_1, Q_1, V_1 are initial density, discharge & velocity respectively at outlet of nozzel & ρ_2, Q_2, V_2 are final density, discharge & velocity respectively at plate

Here
$$\rho_1 = \rho_2 = 1000\text{ kg/m}^3$$
$$Q_1 = 15\text{ lt/s} = 15 \times 10^{-3}\text{ m}^3/\text{s}$$
$$V_1 = \frac{Q_1}{A_1} = \frac{15 \times 10^{-3}}{\frac{\pi}{4} \times (0.03)^2} = 21.22\text{ m/s}$$

(A_1 is area of outlet of Nozzel)

$$Q_2 = 15 \times 10^{-3}\text{ m}^3/\text{s}$$
$$V_2 = 0$$

(water strikes on plate & looses its velocity)

So
$$\text{Force } F = \rho_1 Q_1 V_1 - \rho_2 Q_2 V_2 = \rho Q(V_1 - V_2)$$
$$= 1000 \times 15 \times 10^{-3}(21.22 - 0)$$
$$= 318.30\text{ N}$$

Q. 42 A venturimeter having a throat diameter of 0.1 m is used to estimate the flow rate of a horizontal pipe having a diameter of 0.2 m. For an observed pressure difference of 2 m of water head and coefficient of discharge equal to unity, assuming that the energy losses are negligible, the flow rate (in m^3/s) through the pipe is approximately equal to

(A) 0.500 (B) 0.150

(C) 0.050 (D) 0.015

Sol. 42 Correct option is (C).

Discharge of flow through a venturimeter is given by

$$q = \frac{C_d \times a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Where pressure difference or head, $h = 2\text{ m}$, C_d is coefficient of discharge $C_d = 1$.

a_1 is pipe cross sectional area

So,
$$a_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4} \times (0.2)^2 = 0.031\text{ m}^2$$

a_2 is throat cross sectional area

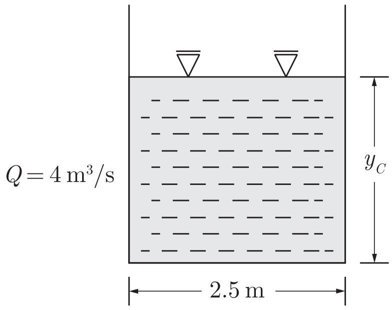
So,
$$a_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4} \times (0.1)^2$$
$$a_2 = 0.0078\text{ m}^2$$
$$g = 9.81\text{ m}^2/\text{s}$$

So
$$q = \frac{1 \times 0.031 \times 0.0078 \times \sqrt{2 \times 9.81 \times 2}}{\sqrt{0.031^2 - 0.0078^2}}$$
$$= 0.050\text{ m}^3/\text{s}$$

Q. 43 A rectangular channel of 2.5 m width is carrying a discharge of $4\text{ m}^3/\text{s}$. Considering that acceleration due to gravity as 9.81 m/s^2, the velocity of flow (in m/s) corresponding to the critical depth (at which the specific energy is minimum) is _______.

Sol. 43 Correct answer is 2.50

Given a rectangular channel with discharge, $Q = 4\,\text{m}^3/\text{s}$ and width, $b = 2.5\,\text{m}$



Now discharge per unit width

$$'q' = \frac{Q}{b} = \frac{4}{2.5} = 1.6\,\text{m}^2/\text{s}$$

Critical depth y_C is given by

$$y_C = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.6^2}{9.8}\right)^{1/3} = 0.639\,\text{m}$$

For depth $= y_C$ & width $= 2.5\,\text{m}$

Let velocity at section be V_m/s

$$\text{discharge} = 4\,\text{m}^3/\text{s (by continuity)}$$

So, $$AV = Q$$

$$V = \frac{4}{(0.639 \times 2.5)} = 2.50\,\text{m/s}$$

Q. 44 Irrigation water is to be provided to a crop in a field to bring the moisture content of the soil from the existing 18% to the field capacity of the soil at 28%. The effective root zone of the crop is 70 cm. If the densities of the soil and water are $1.3\,\text{g/cm}^3$ and $1.0\,\text{g/cm}^3$ respectively, the depth of irrigation water (in mm) required for irrigating the crop is _____

Sol. 44 Correct answer is 91

Depth of water required to be supplied is given by

$$d_w = \frac{\gamma_d\, d(F_C - m_0)}{\gamma_w}$$

d = depth of root zone = 70 cm

γ_d = unit weight of soil $= \rho_d \times g = 1.3g$ $\quad(\rho_d = 1.3\,\text{g/cm}^3)$

γ_w = unit weight of water $= \rho_w \times g = 1g$ $\quad(\rho_w = 1\,\text{g/cm}^3)$

F_C = Field capacity of soil = 28% = 0.28

(g is acceleration due to gravity)

m_0 = existing moisture content = 18% = 0.18

So $$d_w = \frac{1.39g \times 70 \times (0.28 - 0.18)}{g \times 1} = 9.1\,\text{cm}$$

Expressing d_w in mm, $d_w = 91\,\text{mm}$.

Q. 45 With reference to a standard Cartesian (x, y) plane, the parabolic velocity distribution profile of fully developed laminar flow in x-direction between two parallel, stationary and identical plates that are separated by distance, h, is given by the expression

$$u = -\frac{h^2}{8\mu}\frac{dp}{dx}\left[1 - 4\left(\frac{y}{h}\right)^2\right]$$

In this equation, the $y = 0$ axis lies equidistant between the plates at a distance $h/2$ from the two plates, p is the pressure variable and μ is the dynamic viscosity term. The maximum and average velocities are, respectively

(A) $u_{\max} = -\frac{h^2}{8\mu}\frac{dp}{dx}$ and $u_{\text{average}} = \frac{2}{3}u_{\max}$

(B) $u_{\max} = \frac{h^2}{8\mu}\frac{dp}{dx}$ and $u_{\text{average}} = \frac{2}{3}u_{\max}$

(C) $u_{\max} = -\frac{h^2}{8\mu}\frac{dp}{dx}$ and $u_{\text{average}} = \frac{3}{8}u_{\max}$

(D) $u_{\max} = \frac{h^2}{8\mu}\frac{dp}{dx}$ and $u_{\text{average}} = \frac{3}{8}u_{\max}$

Sol. 45 Correct option is (A).

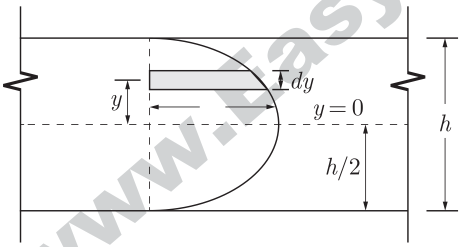
Given parabolic velocity distribution

$$U = -\frac{h^2}{8\mu}\frac{dP}{dx}\left[1 - 4\left(\frac{y}{h}\right)^2\right]$$

$y = 0$ to $y = h/2$ (where $h =$ distance b/w plates)

by putting $y = 0$ we get maximum value of U.

So maximum velocity $U_{\max} = -\frac{h^2}{8\mu}\frac{dP}{dx}\left[1 - 4\left(\frac{0}{h}\right)^2\right] = -\frac{h^2}{8\mu}\frac{dP}{dx}$ ...(1)



Taking an element of dy thickness and unit width at a distance of y from centre in velocity profile.

discharge $$Q_{av} = \int U dA = 2\int_0^{h/2} -\frac{h^2}{8\mu}\frac{dP}{dx}\left[1 - \frac{4y^2}{h^2}\right](dy \times 1)$$

$$= -\frac{h^2}{4\mu}\frac{dP}{dx}\left[y - \frac{4y^3}{3h^2}\right]_0^{h/2} = -\frac{h^2}{4\mu}\frac{dP}{dx}\left[\frac{h}{2} - \frac{h}{6}\right]$$

$$= -\frac{h^3}{12\mu}\frac{dP}{dx}$$

$U_{av} =$ average velocity $$= \frac{Q_{av}}{\int dA} = \frac{Q_{av}}{2\int_0^{h/2} dy \times 1} = \frac{Q_{av}}{h} = -\frac{h^2}{12\mu}\frac{dP}{dx}$$ ...(ii)

From (1) & (2)

$$U_{av} = \frac{2}{3}U_{\max}$$

Q. 46 A suspension of sand like particles in water with particles of diameter 0.10 mm and below is flowing into a settling tank at $0.10\ \text{m}^3/\text{s}$. Assume $g = 9.81\ \text{m/s}^2$, specific gravity of particles $= 2.65$, and kinematic viscosity of water $= 1.0105 \times 10^{-2}\ \text{cm}^2/\text{s}$. The minimum surface area (in m^2) required for this settling tank to remove particles of size 0.06 mm and above with 100% efficiency is ______.

Sol. 46 Correct answer is 31.25

Given S.G. of particles $= 2.65 = G$

$Q =$ discharge of flow into settling tank $= 0.10\,\text{m}^3/\text{s}$

Kinematic viscosity $= \nu = 1.0105 \times 10^{-2}\,\text{cm}^2/\text{s} = 1.0105 \times 10^{-6}\,\text{m}^2/\text{s}$

Particle size to be removed,

$$d = 0.06\,\text{mm} = 0.06 \times 10^{-3}\,\text{m}$$

So. by stokes law settling velocity for the particles is given by

$$V = \frac{g}{18}\frac{(G-1)d^2}{\nu}$$

$$= \frac{9.81 \times (2.65 - 1)(0.06 \times 10^{-3})^2}{18 \times (1.0105 \times 10^{-6})}$$

So $\quad V = 3.20 \times 10^{-3}\,\text{m/s}$

So overflow rate $\quad V = \frac{Q}{A}$ (where A is area of settling tank)

So $\quad A = \frac{Q}{V} = \frac{0.10\,\text{m}^3/\text{s}}{3.20 \times 10^{-3}\,\text{m/s}} = 31.25\,\text{m}^2$

So $\quad \text{Area} = 31.25\,\text{m}^2$

Q. 47 A surface water treatment plant operates round the clock with a flow rate of $35\,\text{m}^3/\text{min}$. The water temperature is 15°C and jar testing indicated an alum dosage of $25\,\text{mg/l}$ with flocculation at a Gt value of 4×10^4 producing optimal results. The alum quantity required for 30 days (in kg) of operation of the plant is _____.

Sol. 47 Correct answer is 37800.

Given $\quad$ Flow rate $= 35\,\text{m}^3/\text{min} = 35 \times 10^3\,\text{lt/min}$

Alum dosage $= 25\,\text{mg/lt}$

Time $= 30\,\text{days} = 30 \times 60 \times 24\,\text{min}$

So

Alum dosage in mg for 30 days

$$`M' = (35 \times 10^3) \times (25) \times (30 \times 60 \times 24)$$

$$`M' = 3.78 \times 10^{10}\,\text{mg}$$

Dosage in kg, $M = 3.78 \times 10^{10} \times 10^{-6} = 37800\,\text{kg}$

G_t value is nowhere to be used.

Q. 48 An effluent at a flow rate of $2670\,\text{m}^3/\text{d}$ from a sewage treatment plant is to be disinfected. The laboratory data of disinfection studies with a chlorine dosage of $15\,\text{mg/l}$ yield the model $N_t = N_0 e^{-0.145t}$ where $N_t =$ number of micro-organisms surviving at time t (in min) and $N_0 =$ number of micro-organisms present initially (at $t = 0$). The volume of disinfection unit (in m^3) required to achieve a 98% kill of micro-organisms is _____

Sol. 48 Correct answer is 50

Disinfection studies with $15\,\text{m/lt}$ chlorine dosage yielded the model

$$N_t = N_0 e^{-0.145t} \qquad ...(1)$$

where $\quad N_t =$ number of micro organism remaining

$N_0 =$ initial number of micro organism

So if 98% micro organism are killed, only 2% are remaining

So if Initially 100 micro organism were there i.e. $N_0 = 100$

So $N_t = 2$

Suppose this reduction from 100 to 2% micro organism took 't' min Using (1)

$$2 = 100e^{-0.145t}$$

$$\ln\frac{2}{100} = (\ln e)\times(-0.145\times t)$$

$$-3.91 = 1\times(-0.145\times t)$$

So $$t = 26.96\text{ mm} = \frac{26.96}{60\times 24}\text{ days}$$

$Q =$ efficient flow rate $= 2670\,\text{m}^3/\text{days}$

So Volume of unit required $= Q\times t = 2670\times\frac{26.96}{60\times 24} = 49.98\,\text{m}^3$

So Volume of unit $\simeq 50\,\text{m}^3$

Q. 49 A waste water stream (flow $= 2\,\text{m}^3/\text{s}$, ultimate BOD $= 90\,\text{mg/l}$) is joining a small river (flow $= 12\,\text{m}^3/\text{s}$, ultimate BOD $= 5\,\text{mg/l}$). Both water streams get mixed up instantaneously. Cross-sectional area of the river is $50\,\text{m}^2$. Assuming the de-oxygenation rate constant, $k' = 0.25/\text{day}$, the BOD (in mg/l) of the river water, 10 km downstream of the mixing point is

(A) 1.68 (B) 12.63

(C) 15.46 (D) 1.37

Sol. 49 Correct option is (C).

When a wastewater with Q_w discharge & L_w, BOD mixes with fresh water stream with Q_s discharge & L_s, BOD. Resultant BOD is given by

$$BOD_{\text{avg}} = L_0 = \frac{Q_w L_w + Q_s L_s}{Q_w + Q_s}$$

Here $Q_w = 2\,\text{m}^3/\text{s};\ Q_s = 12\,\text{m}^3/\text{s};\ L_w = 90\,\text{mg/lt};\ L_s = 5\,\text{mg/lt}$

So $$L_0 = \frac{2\times 90 + 12\times 5}{2+12} = 17.14\,\text{mg/lt}$$

Now lets suppose the mixture moves to downstream with combined discharge $Q_w + Q_s = 14\,\text{m}^3/\text{s}$.

Cross sectional area of rive $= 50\,\text{m}^2$

So Velocity of flow

$$= \frac{14}{50} = 0.28\,\text{m/s} = 0.28\times\frac{18}{5} = 1.008\,\text{km/hr}$$

$t =$ time required to reach 10 km downstream

$$= \frac{10}{1.008} = 9.92\,\text{hr} = \frac{9.92}{24} = 0.413\text{ days}$$

Given Deoxygenation constant $K = 0.25/\text{day}$

Remaining BOD $L = L_0 e^{-kt}$

$$L = 17.14\times e^{-0.25\times 0.413} = 15.458\,\text{mg/lt}$$

Q. 50 In a Marshall sample, the bulk specific gravity of mix and aggregates are 2.324 and 2.546 respectively. The sample includes 5% of bitumen (by total weight of mix) of specific gravity 1.10. The theoretical maximum specific gravity of mix is 2.441. The void filled with bitumen (VFB) in the Marshall sample (in%) is _____

Sol. 50 Correct answer is 67.7

Given

Theoritical maximum specific gravity

$$G_t = 2.441$$

$$\text{S.G. of mixture } G_m = 2.324$$

$$V_a = \text{Volume of air voids} = \left(\frac{G_t - G_m}{G_m}\right) \times 100 = 5.03\%$$

Bitumen by weight is 5% with SG of 1.1

$V_b =$ volume of voids filled with bitumen

So $$V_b = 2.324 \times \frac{5}{1.1} = 10.56\%$$

So $$\text{total \% voids, } V = V_a + V_b = 10.56 + 5.03 = 15.59\%$$

$$\text{Voids filled with Bitumen, } VFB = \frac{V_b}{V_a + V_b} \times 100 = \frac{10.56}{15.59} \times 100$$

$$= 67.7\%$$

Q. 51 A student riding a bicycle on a 5 km one way street takes 40 minutes to reach home. The student stopped for 15 minutes during this ride. 60 vehicles overtook the student (assume the number of vehicles overtaken by the student is zero) during the ride and 45 vehicles while the student stopped. The speed of vehicle stream on that road (in km/hr) is

(A) 7.5 (B) 12

(C) 40 (D) 60

Sol. 51 Correct option is (D).

Given student travels 5 km in 40 min & b/w that he stops for 15 min.

So, $$\text{Average running speed} = \frac{(5)}{(40-15)/60} = 12 \text{ km/hr} \qquad \text{(for bicycle)}$$

Traffic density is number of vehicles per unit length of road. Given 60 vehicle overtook bicycle when it was in motion for $(40-15) = 25$ min & 45 overtook when it was stationary for 15 min. Traffic density for moving as well as stationary condition remain same. Let x be the speed of overtaking vehicle stream.

Traffic density in bicycle's motion

$$K_1 = \frac{\dfrac{60 \text{ veh}}{\left(\frac{25}{60}\right)\text{hr}}}{(x-12)\text{ km/hr}} \qquad \text{...(i)}$$

$q = Ku$

$q =$ Volume

$K =$ density of vehicle

$U =$ relative velocity

Traffic density when bicycle stops for 15 min

$$K_2 = \frac{\dfrac{45 \text{ veh}}{\left(\frac{15}{60}\right)\text{hr}}}{(x-0)} \qquad \text{...(ii)}$$

So (i) = (ii)

$$K_1 = K_2 \Rightarrow \frac{(60/25)}{(x-12)} = \frac{(45/15)}{x}$$

$$x = 60 \text{ km/hr}$$

Q. 52 On a section of a highway the speed density relationship is linear and is given by $v = \left[80 - \frac{2}{3}k\right]$; where v is in km/h and k is in veh/km. The capacity (in veh/h) of this section of the highway would be

(A) 1200 (B) 2400

(C) 4800 (D) 9600

Sol. 52 Correct option is (B).

Given speed density relationship

$$V = \left[80 - \frac{2}{3}K\right]$$

V is in Km/hr & K is density in veh/km

So traffic volume, $q\,(\text{veh/hr}) = KV = 80K - \frac{2K^2}{3}$ ...(i)

Traffic capacity is the maximum volume or flow and can be obtained by differentiating (i) or

$$\frac{dq}{dK} = 0$$

$$\frac{dq}{dK} = 80 - \frac{4K}{3} = 0$$

$$K_{max} = 60 \text{ veh/km}$$

It is density for maximum flow velocity

$$V_{max} = 80 - \frac{2}{3} \times 60 = 40 \text{ km/hr}$$

So capacity flow $q_{max} = K_{max} \times V_{max} = 60 \times 40 = 2400$ veh/hr

Q. 53 A pre-timed four phase signal has critical lane flow rate for the first three phases as 200, 187 and 210 veh/hr with saturation flow rate of 1800 veh/hr/lane for all phases. The lost time is given as 4 seconds for each phase. If the cycle length is 60 seconds, the effective green time (in seconds) of the fourth phase is ________

Sol. 53 Correct answer is 15.82

Average cycle length "C_0" is given by webster method as

$$C_0 = \frac{1.5L + 5}{1 - \Sigma y} \quad \text{...(i)}$$

L is total time lost per phase

So $L = 4 \times 4 = 16$ sec

(its 4 phase signal with 4 sec loss time each phase)

$\Sigma y =$ summation of lane volume each phase/saturation flow

So $\Sigma y = y_1 + y_2 + y_3 + y_4$

where y_1, y_2, y_3 are values of 3 phases & y_4 is value for Green phase

So $y_1 = \frac{200}{1800} = 0.111;\ y_2 = \frac{187}{1800} = 0.1038;\ y_3 = \frac{210}{1800} = 0.116;\ y_4 = ?$

Now $\Sigma y = y_1 + y_2 + y_3 + y_4 = 0.3308 + y_4$ (saturation flow = 1800 Veh/hr)

Using (1) Given cycle length $C_0 = 60$ sec

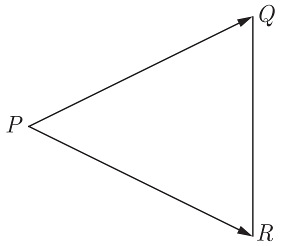
$$60 = \frac{1.5 \times 16 + 5}{1 - (0.3308 + y_4)}$$

$$y_4 = 0.1858$$

So Green phase time $G_0 = \frac{(C_0 - L) \times y_4}{\Sigma y} = \frac{(60 - 16) \times 0.1858}{0.3308 + 0.1858}$

Effective green time $G_0 = 15.82$ sec

Q. 54 A tacheometer was placed at point P to estimate the horizontal distances PQ and PR. The corresponding stadia intercepts with the telescope kept horizontal, are 0.320 m and 0.210 m, respectively. The $\angle QPR$ is measured to be $61^\circ 30' 30''$. If the stadia multiplication constant = 100 and stadia addition constant = 0.10 m, the horizontal distance (in m) between the points Q and R is _____



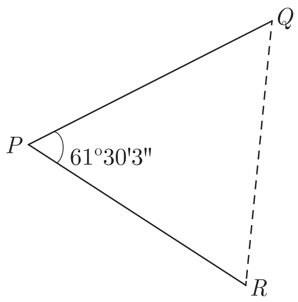
Sol. 54 Correct answer is 28.79

Horizontal distance measured by tacheometer is given as

$$D = Ks + C$$

K is multiplicative constant; s is stadia intercept; C is addition constant

Given here $K = 100$ & $C = 0.10$ m



So $D = 100s + 0.10$

For PQ; $s = 0.320$

So $D(PQ) = 100 \times 0.320 + 0.10 = 32.1$ m

For QR; $s = 0.210$

So $D(PQ) = 100 \times 0.210 + 0.10 = 21.1$ m

Now by using cosine formula

$$QR^2 = PR^2 + PQ^2 - 2PR \times PQ\cos\theta$$

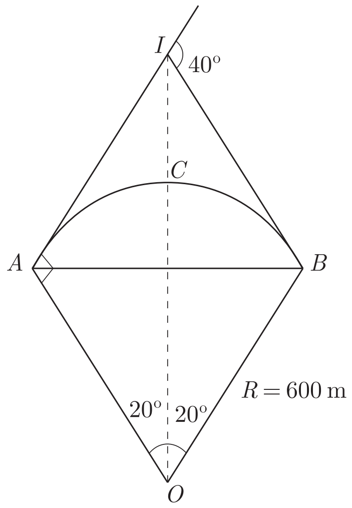
So $$QR = \sqrt{21.1^2 + 32.1^2 - 2 \times 32.1 \times 21.1 \times \cos 61^\circ 30' 30''}$$
$$= 28.79 \text{ m}$$

Q. 55 The chainage of the intersection point of two straights is 1585.60 m and the angle of intersections is 140°. If the radius of a circular curve is 600.00 m, the tangent distance (in m) and length of the curve (in m), respectively are

(A) 418.88 and 1466.08
(B) 218.38 and 1648.49
(C) 218.38 and 415.88
(D) 418.88 and 218.38

Sol. 55 Correct option is (C).

Angle of intersection given is 140° but it should be 40°



Given radius of curve

$$ACB = 600\,\text{m};\ \angle AOB = 40^\circ$$

by Using angle $= \dfrac{\text{arc}}{\text{radius}}$

$$\text{Length of curve } \widehat{ACB} = 40 \times \left(\frac{\pi}{180}\right) \times 600 = 418.66\,\text{m}$$

$$\text{Length of Tangent, } AI \text{ or } IB = R \times \tan(\theta/2) = 600 \times \tan(40/2)$$

$$= 600 \times \tan 20 = 218.38\,\text{m}$$

****

ANSWER KEY

General Aptitude									
1	2	3	4	5	6	7	8	9	10
(D)	(A)	(C)	(25)	(A)	(A)	(C)	(D)	(C)	(0.48-0.49)

Civil Engineering									
1	2	3	4	5	6	7	8	9	10
(B)	(88)	(B)	(D)	(B)	(B)	(B)	(40)	(D)	(3.5)
11	12	13	14	15	16	17	18	19	20
(B)	(0)	(A)	(D)	(D)	(81.3)	(D)	(D)	(D)	(D)
21	22	23	24	25	26	27	28	29	30
(A)	(C)	(20)	(D)	(D)	(A)	(0.27)	(2)	(3)	(B)
31	32	33	34	35	36	37	38	39	40
(2.43)	(50)	(C)	(C)	(B)	(5)	(23.73)	(A)	(6171.356)	(B)
41	42	43	44	45	46	47	48	49	50
(318.30)	(C)	(2.50)	(91)	(A)	(31.25)	(37800)	(50)	(C)	(67.7)
51	52	53	54	55					
(D)	(B)	(15.82)	(28.79)	(C)					